



Congestion income allocation under Flow-Based Market Coupling

CWE Market Coupling

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1 Introduction

The sharing of the congestion income under Flow-Based Market Coupling between the hubs of the CWE (Central-Western Europe) region is described in this document. This description is only valid for the standard hybrid coupling method. The treatment of resale costs resulting from the 'Use It Or Sell It' (UIOSI) mechanism for Long-Term Capacity Rights, currently in place under ATC MC, is integral part of the methodology as it will remain also under FB MC.

Please note that the results of the sharing key will be monitored during 2014 and 2015 (beginning of the year). The TSOs will provide a report to the NRAs by the end of 2015 containing an evaluation of the sharing key of the external pot with a recommendation either to remain to the chosen sharing key or to justify a new sharing key for the external pot in case unforeseen effects are identified.

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2 General definitions

The overall congestion income can be calculated by the following formula:

$$CI = - \sum_{i=1}^{NH} \text{netPOS}_i \times CP_i \quad (\text{Eq. 1})$$

Where:

netPOS_i: net position of hub i

CP_i: clearing price of hub i

NH: total number of hubs

The impact of commercial flows on the critical branches (CB) is given by the power transfer distribution factors which are organized in the so-called PTDF-Matrix. This matrix translates the net positions into physical flows on the critical branches. Hence, the additional flow AAF_i associated to network constraint i can be calculated by multiplying the according power transfer distribution factor PTDF_{i,j}, where j refers to the respective hub, by the net hub position, using the following equation:

$$AAF_i = \sum_{j=1}^{NH} \text{PTDF}_{i,j} \times \text{netPOS}_j \quad (\text{Eq. 2})$$

Where:

PTDF_{i,j}: power transfer distribution factor of hub j on critical branch i

netPOS_j: net position of hub j

NH: total number of hubs

Definition of shadow price

In mathematical terms, the Flow-Based Market Coupling (FBMC) algorithm is an optimization procedure that generates so-called shadow prices on every Flow-Based (FB) constraint, i.e. on each modelled network element that is monitored under certain operational conditions (such as outages).

The shadow price represents the marginal increase of the objective function (DA market welfare) if the constraint is marginally relaxed. In other words: the shadow price is a good indication of the increase in DA market welfare that would be induced by an increase of capacity on the active network constraint. As a consequence, non-binding network constraints in the market coupling solution have a shadow price of zero, since an increase of capacity on those network elements would neither change the optimal market coupling solution nor the flow on the network element concerned.

The overall congestion income for flow-based market coupling can therefore also/alternatively be calculated on the basis of the shadow prices (SP) and the flows induced by the net positions resulting from the market coupling as well, using the expression

$$CI = \sum_{i=1}^{NC} AAF_i \times SP_i \quad (\text{Eq. 3})$$

Where:

SP_i : shadow price associated to network constraint i

AAF_i : additional aggregated flow associated to network constraint i

NC : total number of network constraints

Hence, equation (Eq. 3) represents the mathematical equivalent to equation (Eq. 1).

The results of the FBMC on January 3, 2013, 09.00-10.00 is shown in Figure 1. The same example is used throughout the document except in Chapter 10.

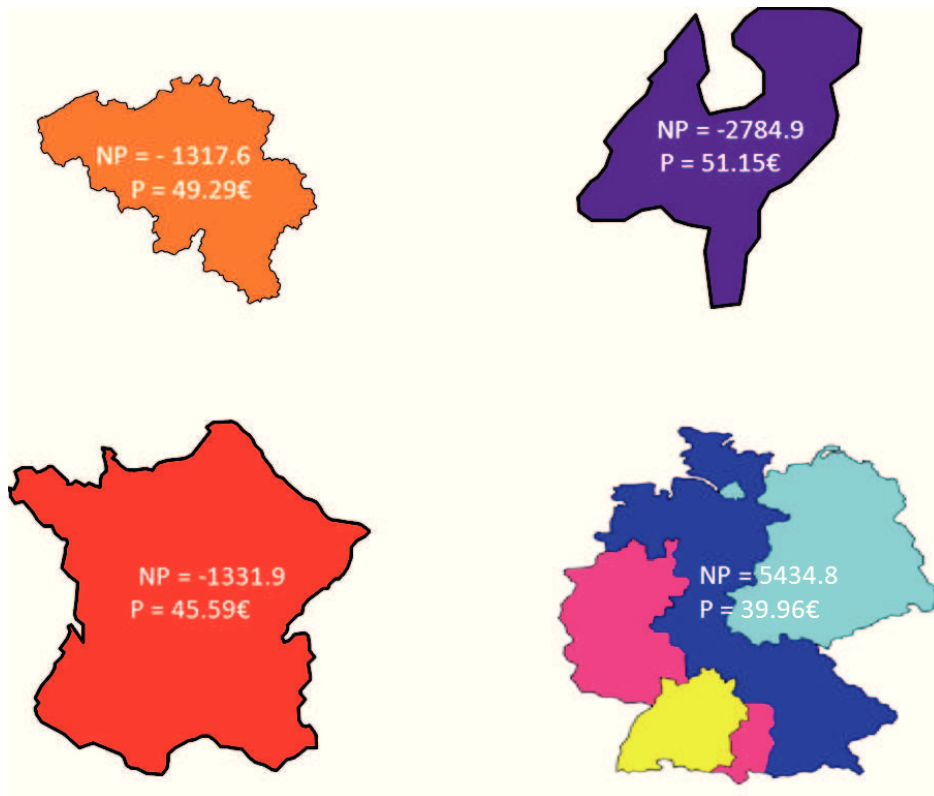


Figure 1: Market Results of FBMC on the 3rd of January 2013, for 09h00 – 10h00

Due to rounding, the sum of the net positions of the hubs does not sum up to zero. The simulated FBMC was constrained by one critical branch, having a shadow price:

CB	RAM ¹ (MW)	shadow price (€/MWh)
CB1	575.3	88.69

The total congestion income equals:

$$CI = 575.3 \times 88.69 = 51\,023.40\text{€}$$

From the net positions and prices we can obtain the congestion income as well:

$$CI = -1317.6 \times 49.29 - 1331.9 \times 45.59 + 5434.8 \times 39.96 - 2784.9 \times 51.15 = 50\,938.90\text{€}$$

¹ In case of congestion, the Remaining Available Margin (RAM) is equal to the additional aggregated flow associated to the Network Constraint

The numbers are equivalent to one another, besides the €84.50 difference due to rounding effects in the numbers.

3 Criteria for sharing income

The qualitative criteria are depicted below in more detail.

3.1 Short & Long Term Incentive compatible

According to Article 6.1 of Annex 1 to EU Regulation 2009/714/EG the procedure for the distribution of congestion income shall not provide a disincentive to either reduce congestion nor to distort the allocation process in favour of any party requesting capacity or energy.

Objectives: Efficient use of existing and efficient investments in transmission assets.

3.2 Transparent and easy to understand

Objectives: The distribution of congestion income should be transparent and auditable, which means that very complex sharing keys are not preferred. It should be easy to show in which way the congestion income is shared by the hubs and how this is integrated in the total picture of the congestion income cycle.

3.3 Robustness against gaming

Objectives: The sharing key should not give room for optimisation of any individual hub's share of the congestion income by gaming on data manipulation.

3.4 Fairness and Non discriminatory

Objectives: The sharing key should be based on elements related to the management of capacity for cross-border transactions.

3.5 Predictability and Limited volatility

Objectives: The sharing key should allow a forecast of the financial outcome and should not lead to a higher volatility of each share compared to the status quo, in order to allow a reasonable financial planning and cash flow management.

3.6 Smoothness of transition

Objectives: the current congestion income distribution should not be changed in a radical way in the short term in order to limit the financial impact on all parties.

3.7 Positive income per hub

Objectives: As long as the long term allocated (LTA) capacity domain is included in the FB domain, the hourly individual net income of each hub remains positive.

3.8 Stability in case of extension

Objectives: The current congestion income distribution for the CWE hubs should not be changed in a radical way when new hubs are joining the FB region.

3.9 Positive Day-ahead market welfare gain compared to ATC MC

Objectives: The DA market welfare (producer surplus + consumer surplus + congestion income) gain for a hub should be positive compared to ATC MC.

Within the process of developing the sharing methodology for the congestion income, these criteria and objectives were taken into account. Therefore, the presented solution is one that fits the criteria best.

4 Nomination proof and additional aggregated flow calculation

The sharing of congestion income and resale costs of each hub is made independent of the actual nomination level on a border by the market participants that hold the long term rights. As such, the sharing key is made 'nomination proof'. To establish this, the hourly resale costs per hub

border are calculated from the total volume of allocated long term rights multiplied by the hourly price difference that occurs on that border, instead of only considering the resold part of the LTA multiplied by the price difference. Furthermore, the net positions to derive the overall congestion income need to be corrected with the Long Term Nominations (LTN), such that the income is shared as if all LTA have not been nominated. These updated net positions are used throughout the whole calculation methodology, except for the calculation of the overall congestion income. The netted long term nominations on the CWE borders, for our example, are shown in Figure 2.

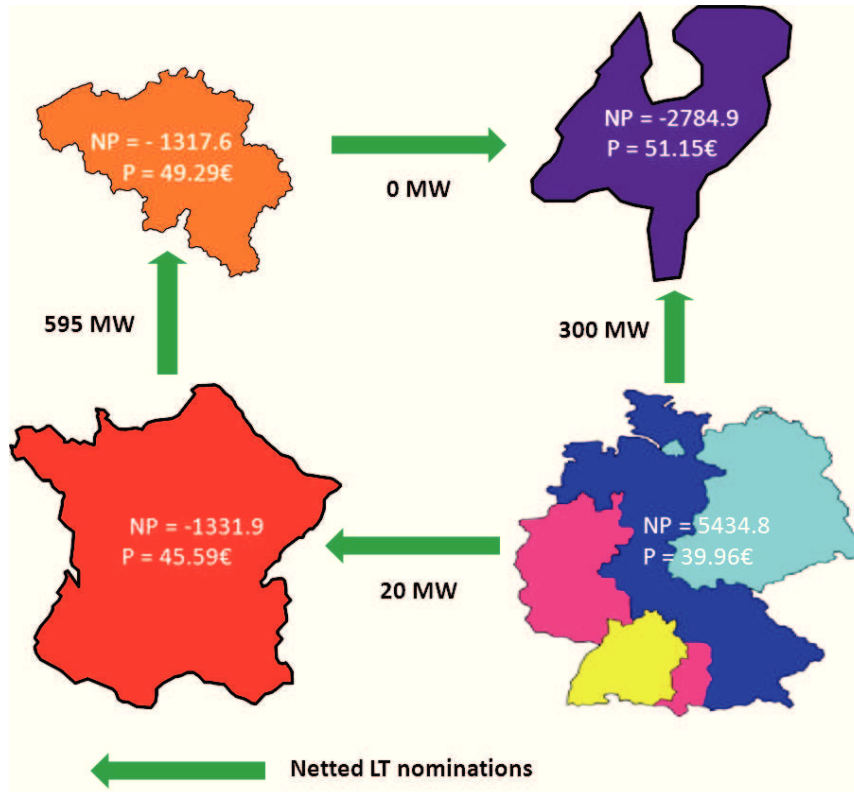


Figure 2: The netted long term nominations on the CWE borders

Since the net positions change, the AAFs change accordingly (Eq. 4), which is an adaptation of the earlier shown equation (Eq. 2). The resulting AAFs for our example are shown in Figure 3. The flows on the critical branches on a border are aggregated on a hub border level.

$$AAF_i = \sum_{j=1}^{NH} PTDf_{i,j} \times netPOS(FBMC + LTN)_j \quad (Eq. 4)$$

Where:

$PTDF_{i,j}$: power transfer distribution factor of hub j on critical branch i

$netPOS_j$: net position of hub j

NH : total number of hubs

$FBMC$: the part of the net position allocated through the daily flow-based market coupling (resold LTA and additional margin provided by the TSOs)

LTN : a correction of the net position due to the level of Long Term Nominations

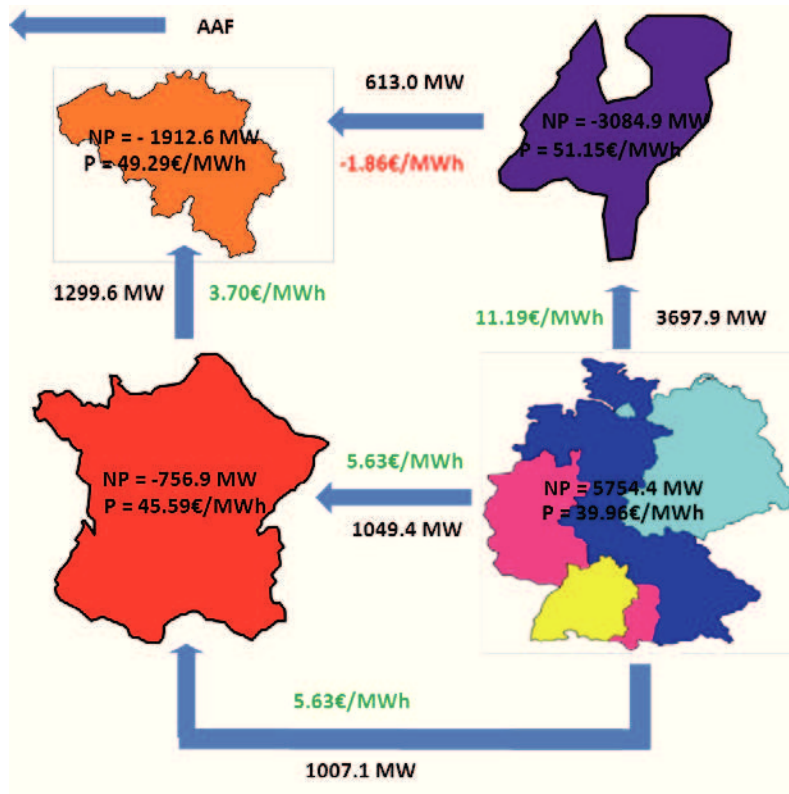


Figure 3: the updated net positions and corresponding additional aggregated flows

5 Cross Border clearing price x market flows absolute (CBCPM abs)

The Congestion Income Allocation mechanism for CWE – which could serve as a blueprint for other FB coupled regions as well – takes up the fundamental characteristics of the well-known ATC scheme. Even though the results of CWE FB MC are hub net positions and clearing prices, the FB sharing key (CBCPM abs) – in a first step – assigns a Border Value to each individual hub-border in order to allocate the congestion income to the respective capacity holders. The idea is to share the congestion income based on economic indicators related to the allocation of cross-border capacity in zonal-markets, i.e. market price differences and allocated cross-border flows. However, the FB sharing key is also in line with the principle of price formation in FB (Eq. 5):

$$\frac{\Delta CP_{hub\ i \rightarrow j}}{\Delta PTD F_{hub\ i \rightarrow j, k}} = Shadow\ Price \geq 0 \quad (Eq. 5)$$

Where:

$\Delta PTD F_{hub\ i \rightarrow j, k}$: power transfer distribution factor difference between hub i and j for critical branch k

$\Delta CP_{hub\ i \rightarrow j}$: clearing price difference between hub i and hub j

$\Delta PTD F$ of the limiting CB is proportional to ΔCP . The $\Delta PTD F$ between the hubs close to the limiting CB is larger than the $\Delta PTD F$ between the hubs far away. Therefore, the price difference between the hubs close to the limiting CB is larger than the price difference between hubs far away.

The aforementioned Border Value is calculated by multiplying the respective AAFs by the price difference of the neighbouring hubs. These Border Values are equally shared between the related hubs. As such, the key is a continuation of the current ATC scheme.

However, in contrast to ATC, under FB MC negative Border Values might occur if AAFs are directed against the clearing price difference (the price difference of the neighbouring hubs is – in the direction of the AAF – negative)². Nevertheless, since also those flows contribute to the maximization of day-ahead market welfare, Border Values are always taken into account in absolute terms. Since the absolute value of the Border Values is taken into account, a rescaling to the original overall congestion income is required.

5.1 Calculations

For the calculation of the CBCPM ABS key, the absolute Border Value per hub is considered as shown below:

$$CI_{Hub_i}^{CBCPM\ ABS} = \frac{1}{2} \times \frac{\sum_{j=1}^{NH} |AAF_{hub\ i \rightarrow j} \times \Delta CP_{hub\ i \rightarrow j}|}{\sum_{i=1}^{NH} \sum_{j>i}^{NH} |AAF_{hub\ i \rightarrow j} \times \Delta CP_{hub\ i \rightarrow j}|} \times CI \quad (\text{Eq. 6})$$

Where:

CI_{Hub_i} : congestion income associated to hub i

$AAF_{hub\ i \rightarrow j}$: sum of additional flows from hub i to hub j

$\Delta CP_{hub\ i \rightarrow j}$: clearing price difference between hub i and hub j

NH : total number of hubs

5.2 Properties of the proposed sharing key

The CBCPM abs sharing key can be seen as an “evolution” of the ATC sharing key principle to rationalize the sharing of congestion income. The basic idea of the CBCPM sharing key is transparency and easiness to understand.

The income is linked to congested CB(s) that set(s) the prices: the $\Delta PTDf$ close to the limiting branch is large and therefore, the price difference is also large. This means a large congestion income on the borders close to the congestion. So the price difference is an indication of the location of the congestion. As such, the congestion income is an indication of the criticality of a congestion.

The sharing key has a good stability in case of extensions. In case a hub with a border with recurrent congestions joins, the congestion income sharing is mainly attributed to that border and vice versa: if a hub without congestion on its borders joins, few congestion incomes will be attributed to this hub.

The absolute variant of the sharing key avoids negative net congestion income on a hub border.

6 Determination of the internal and external pot

As previously mentioned, the total congestion income is related to the shadow prices of the congested critical branches somewhere inside CWE. Using the AAFs, and after having adapted the net positions with the Long-Term nominations, it is possible to divide this global income into an “internal” and an “external” pot.

The internal pot is linked to the internal flows (AAFs) on the CWE internal borders, while the external pot is related to the flows exiting and re-entering the CWE FB area through neighbouring hubs.

² This situation can also occur within FB Intuitive MC, since a situation is defined as intuitive if there exist at least one possible set of intuitive bilateral exchanges. The AAFs resulting from the FBI MC are different from this set of bilateral exchanges.

6.1 Calculation

For the computation of both the internal and external pot, we consider that all flows (AAFs) help to reach the optimum in CWE day-ahead market welfare, whatever the direction of the flow (with or against the price difference). This is in line with the choice of the CBCPM absolute key that was selected. It also ensures that both incomes are positive, which would not always be the case without considering absolute values. This means that we sum up the absolute Border Values for all internal and external hub borders respectively:

- Unscaled Internal pot = $\sum |(AAF(\text{internal hub borders}) \times \Delta P)|$ (Eq. 7)

- Unscaled External pot = $\sum |(AAF(\text{external hub borders}) \times \Delta P)|$ (Eq. 8)

These definitions have several consequences. First of all, the use of absolute values implies that the sum of the two pots may exceed the overall CWE congestion income. When sharing each of the pots, a pro-rata rescaling is then needed to correct this effect as shown in (Eq. 9) and (Eq.10). The second effect is that additional rules have to be defined in case of extensions. The latter is explained in section 10.1.

- $internal\ pot = \frac{unscaled\ internal\ pot \times overall\ CI}{(unscaled\ internal\ pot + unscaled\ external\ pot)}$ (Eq. 9)

- $external\ pot = \frac{unscaled\ external\ pot \times overall\ CI}{(unscaled\ internal\ pot + unscaled\ external\ pot)}$ (Eq.10)

For the sharing of each of the pots, different sharing keys will be used:

- For the sharing of the internal pot, the CBCPM absolute sharing key is used.
- The sharing of the external pot is explained in Chapter 9.

6.2 Example

The updated net positions, market clearing prices and AAFs are shown in Figure 4 (03/01/2013 hour 10):

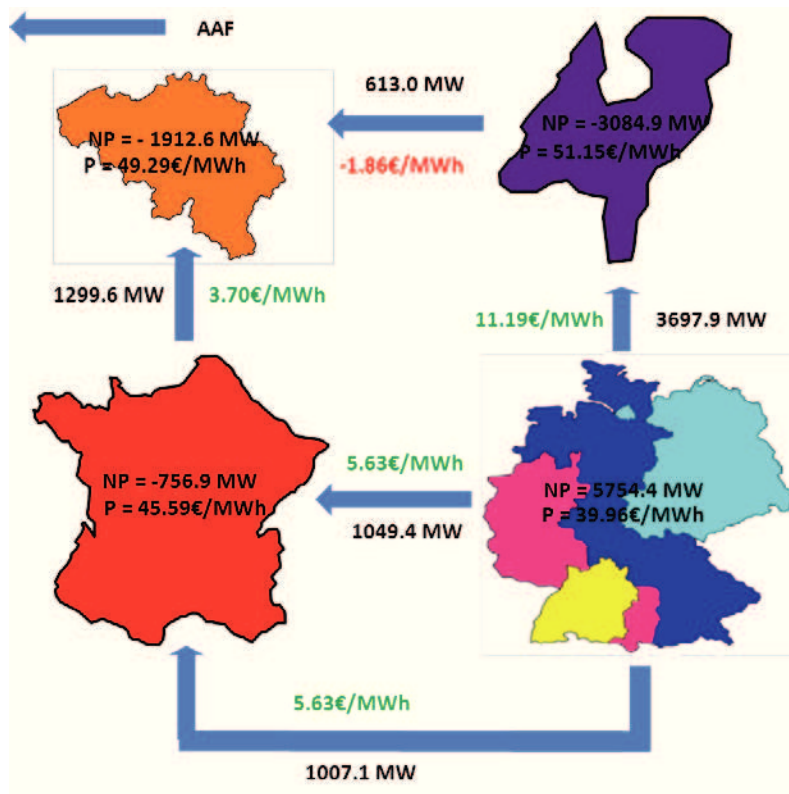


Figure 4: The updated net positions and corresponding additional aggregated flows

Applying these principles to our example leads to these computations (Table 1):

$$\text{Unscaled internal pot} = \sum |(AAF(\text{internal}) \times \Delta P)| = 53\,226.10\text{€}$$

$$\text{Unscaled external pot} = \sum |(AAF(\text{external}) \times \Delta P)| = 5\,669.97\text{€}$$

Border	AAF × ΔP
DE-FR	1049.4 × 5.63 = 5 910.94
DE-NL	3697.9 × 11.19 = 41 379.50
BE-NL	613.0 × 1.86 = 1 140.18
BE-FR	1299.6 × 3.69 = 4 795.52
Sum of absolute Border Values for all internal hub borders => Unscaled internal pot	53 226.10
Sum of absolute Border Values for all external hub borders => Unscaled external pot	1007.1 × 5.63 = 5 669.97

Table 1: Calculation of the border values

As the sum of the unscaled internal pot and unscaled external pot (53 226.10€ + 5 669.97€) exceeds the overall CWE congestion income (50 938.90€), a rescaling is necessary (Table 2):

Border	Congestion Income
DE-FR	$5\,910.94 \times (50\,938.9) / (53\,226.1 + 5\,669.97) = 5\,112.34\text{€}$
DE-NL	$41\,379.5 \times (50\,938.9) / (53\,226.1 + 5\,669.97) = 35\,788.90\text{€}$
BE-NL	$1\,140.18 \times (50\,938.9) / (53\,226.1 + 5\,669.97) = 986.14\text{€}$
BE-FR	$4\,795.52 \times (50\,938.9) / (53\,226.1 + 5\,669.97) = 4\,147.62\text{€}$
Internal pot	46 035.00€
External pot	4 903.93€

Table 2: Calculation of the congestion income on a hub border and the internal and external pot

Internal pot = 46 035.00€

External pot = 4 903.93€

The congestion income on the borders is shown in Figure 5.

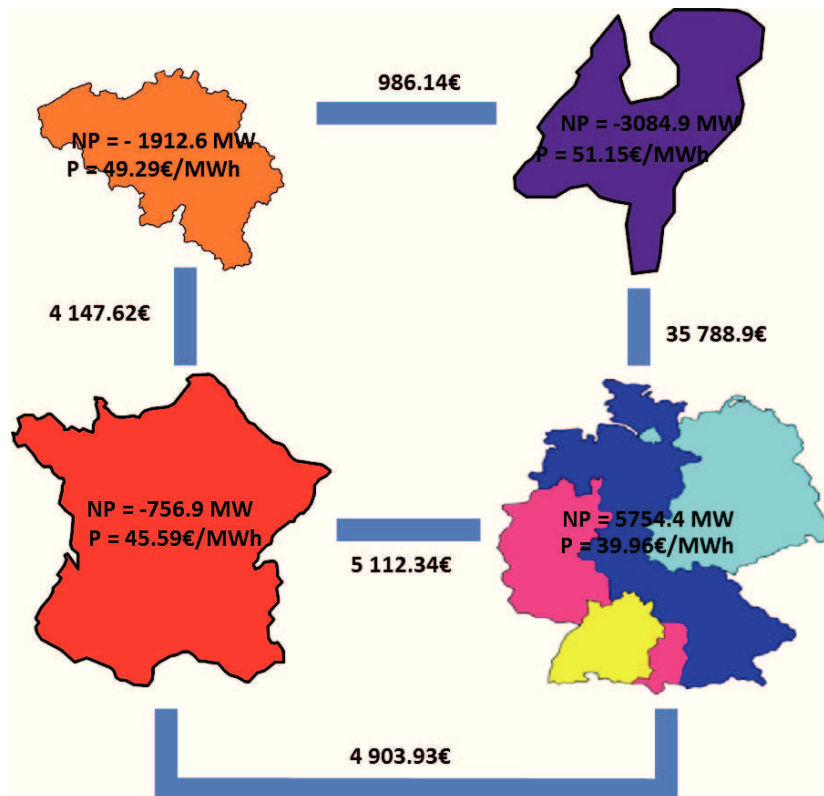


Figure 5: The congestion income on the hub borders

7 Sharing of the hub border income

The congestion income on the hub borders is shared equally between the neighbouring hubs as shown in Figure 6.

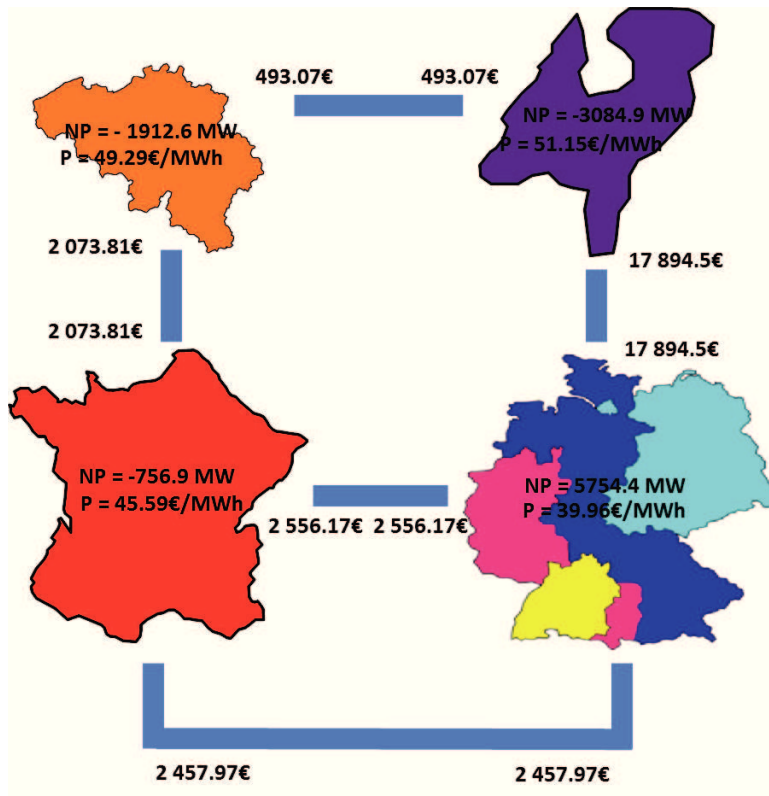


Figure 6: The congestion income per hub border on each side of the border

8 Principles of the resales under Flow-Based MC

8.1 Price of the resales

The TSOs, through the "Use It Or Sell It" principle, enable the Market Participants that acquire some bilateral capacities (based on ATC) in Yearly and Monthly auctions to automatically resell these capacities at the daily allocation in case they do not nominate these capacities. These resales will lead, in ATC but also in Flow-Based, to the payment of the positive price spread between the two hubs multiplied with the volume of capacity resold. The resale costs in Flow-Based can be defined in 2 ways as shown in (Eq. 11) and (Eq.12);

$$Resale\ Cost = \sum_{i,j} (LTA_{i \rightarrow j} - LTN_{i \rightarrow j}) \times \max(0, \Delta CP_{hub\ i \rightarrow j}) \quad (Eq. 11)$$

$$Resale\ Cost = \sum_{NC} AAF_{resale,i} \times SP_i \quad (Eq.12)$$

Where:

- $LTA_{i \rightarrow j}$: long term allocated capacity on the border in the direction from i to j.
- $LTN_{i \rightarrow j}$: long term nominated capacity on the border in the direction from i to j.
- $\Delta CP_{hub\ i \rightarrow j}$: clearing price difference between hub i and hub j
- $AAF_{resale,i}$: positive margin freed by the resales on critical branch i.
- SP_i : shadow price associated to network constraint i
- NC : total number of network constraints

8.2 Maximum cost of the resale accepted by TSOs

As it is already the case in ATC Market Coupling, TSOs propose to reimburse the real cost of resale to the Market Participants without setting a cap on the overall cost of resales.

From (Eq.12), one can see that if the overall margin freed by all resales on each critical branch is lower than the margin made available by the TSOs to the Market Coupling, the congestion income from Flow-Based Market Coupling is higher than the resale cost as shown in Figure 7. We can conclude that if the Long Term ATC domain is included in the Flow-Based domain, the resale costs are covered by the hourly congestion income. The numerical proof that the resale costs are smaller than the overall congestion income when the LTA domain is included in the FB domain can be found in Annex 1.

During the external parallel run, some hours were detected where the total congestion income was not sufficient to cover the resale costs for all hub borders due to the fact that the LTA ATC domain was outside the Flow-Based domain. After the FB go live, CWE TSOs ensure that the capacity provided to the Market Participants through Long Term allocations is included in the Flow-Based Domain. Once this inclusion is ensured, the hourly resale cost will be lower than the hourly congestion income.

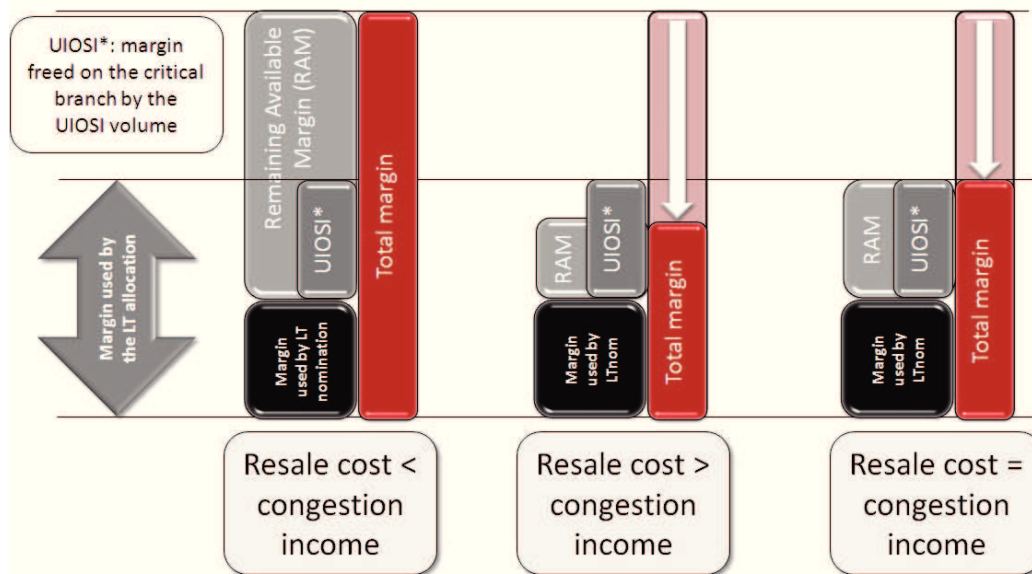


Figure 7: Relationship between overall congestion income, resale cost and margin on a critical branch

8.3 Resale methodology in line with treatment of external pot

Resale costs are based on a scheduled flow. The income sharing key uses physical flow (i.e. AAFs). This can give an inconsistency between the chosen resale methodology and the treatment of the external pot (Chapter 9), because for hubs with open borders the sum of the physical resale flows on the internal borders does not sum up to the total scheduled resale flow on all internal borders. To make the resale methodology in line with the treatment of the external pot, the following principle is applied:

- For a hub with closed borders the resale cost divided by two is assigned to its side of the respective closed border (as $\sum (AAF_{resale}) = \text{resale volume}$).
- For a hub with open borders, the part of the resale cost that is linked to the $\sum(AAF_{resale})$, divided by two, is assigned to its side of the closed border, whereas the part of the resale cost that is linked to the difference between the resale volume and the $\sum(AAF_{resale})$, divided by two, is assigned to the open border.

The methodology is explained below in Figure 8 and Figure 9.

As a consequence, both sides of a border can have a different resale cost (in CWE only for DE-NL and BE-FR).

In Figure 8, an LTA of 614MW between Belgium and the Netherlands is converted in resale flows (AAF_{resale}) via the PTDFs. Belgium and the Netherlands are two hubs with only closed borders. The resale cost on this border is 887.54€³. At the Belgian side, all resale flows are leaving through internal borders. So the cost at this side of the border is half of the resale cost, i.e. 443.77€. The same reasoning can be followed for the Dutch side of the border.

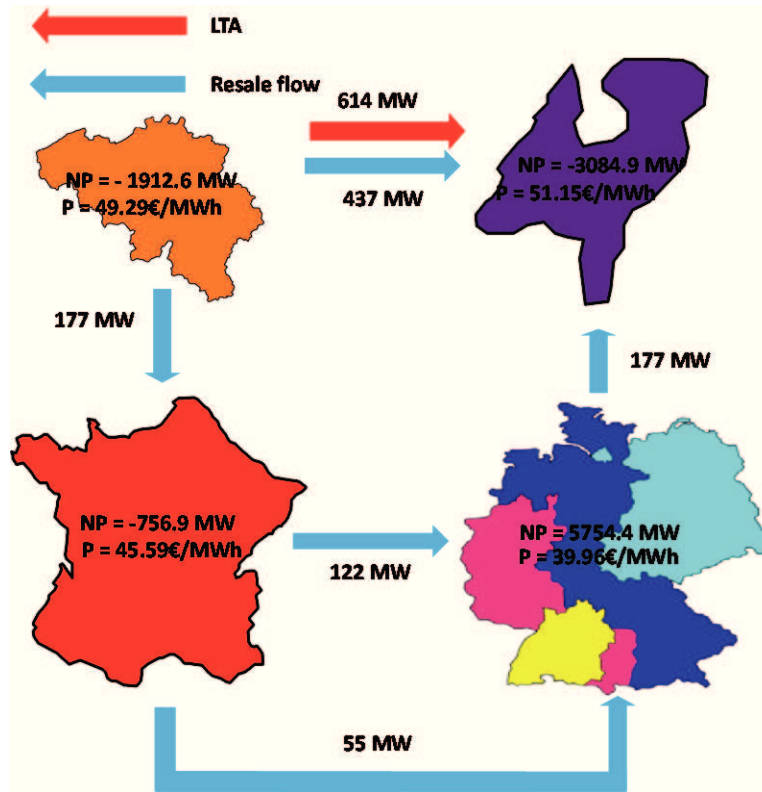


Figure 8: The resale flows for an LTA between Belgium and the Netherlands

In Figure 9, an LTA between Germany and the Netherlands is converted in resale flows via the PTDFs. The Netherlands is a hub with only closed borders, whereas Germany has an open border. The resale cost on this border is 9423.63€⁴. At the Dutch side of the border, all flows are entering at internal borders and therefore, the resale cost at that side of the border is half of the resale cost, i.e. 4711.82€. At the German side, only 963MW of the 1083MW (89%) is leaving the German hub via internal borders, thus only 89% of half of the resale cost is awarded to that side of the border (i.e. 4191.2€). The remaining part (i.e. 520.62€) is awarded to the border Germany – slack zone.

³ Please note that the resale cost has been scaled down in accordance to the overall resale cost (see also section 4)

⁴ Please note that the resale cost has been scaled down in accordance to the overall resale cost (see also section 4)

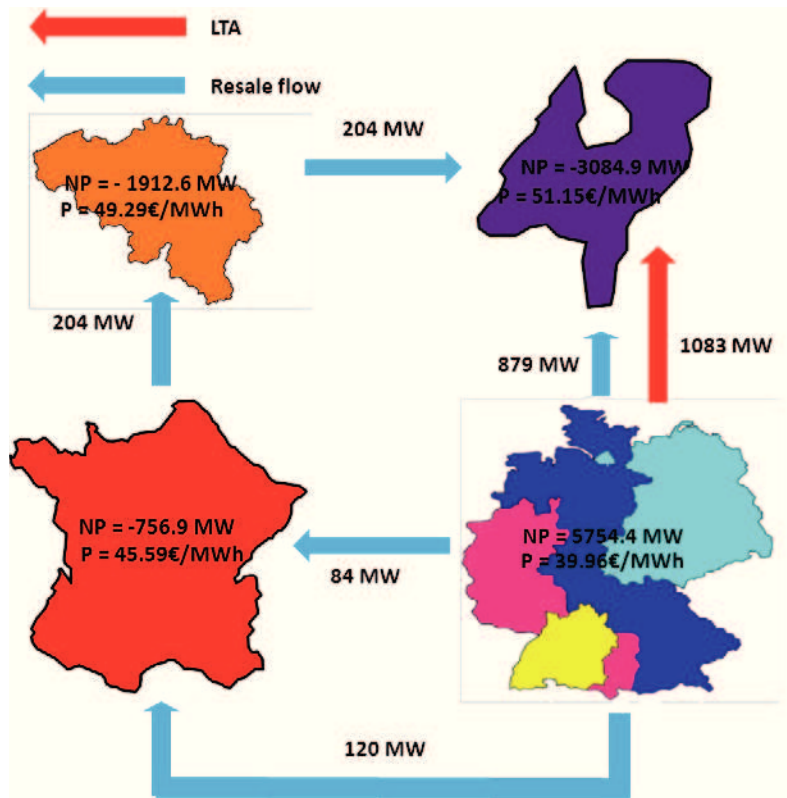


Figure 9: The resale flows for an LTA between Germany and the Netherlands

8.4 Resale methodology with socialization

The resale cost is calculated on a hub border basis; for internal and external borders (see also chapter 9). Each TSO is responsible for compensating the resale costs on its side of the border. The steps to arrive at the resale cost per side of a hub border are reflected in the chart below:

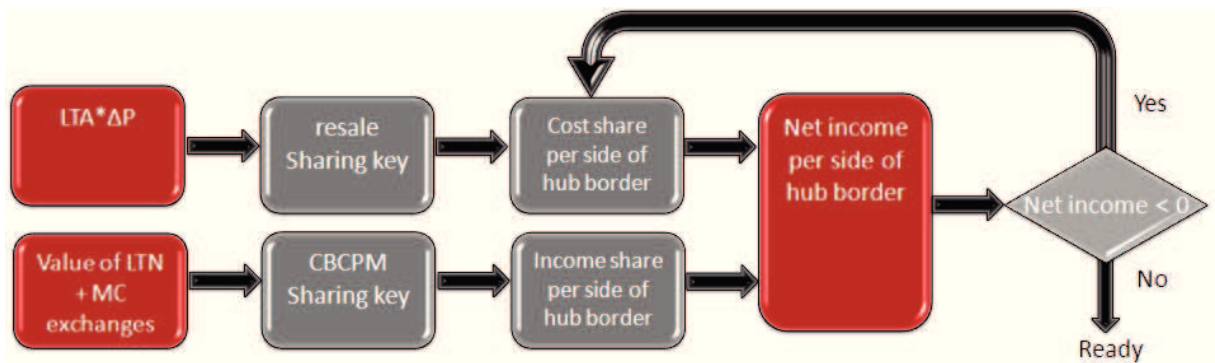


Figure 10: Resale methodology

Figure 11 shows the congestion income per hub border on each side of the hub border and Figure 12 shows the resale costs on each side of the hub border. The difference between these values is the net congestion income per hub border (i.e. income after resales).

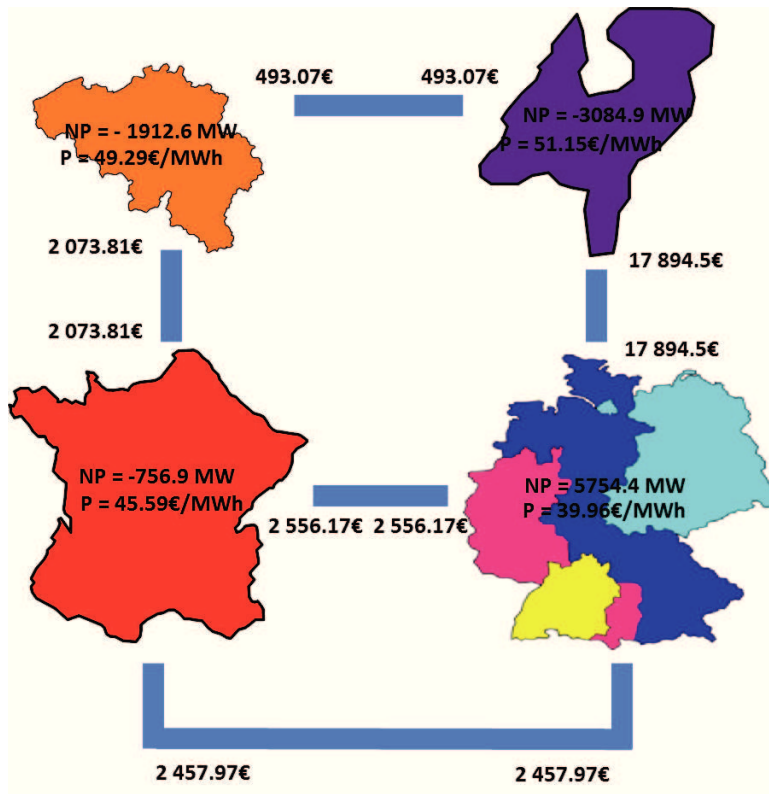


Figure 11: The congestion income per hub border on each side of the border

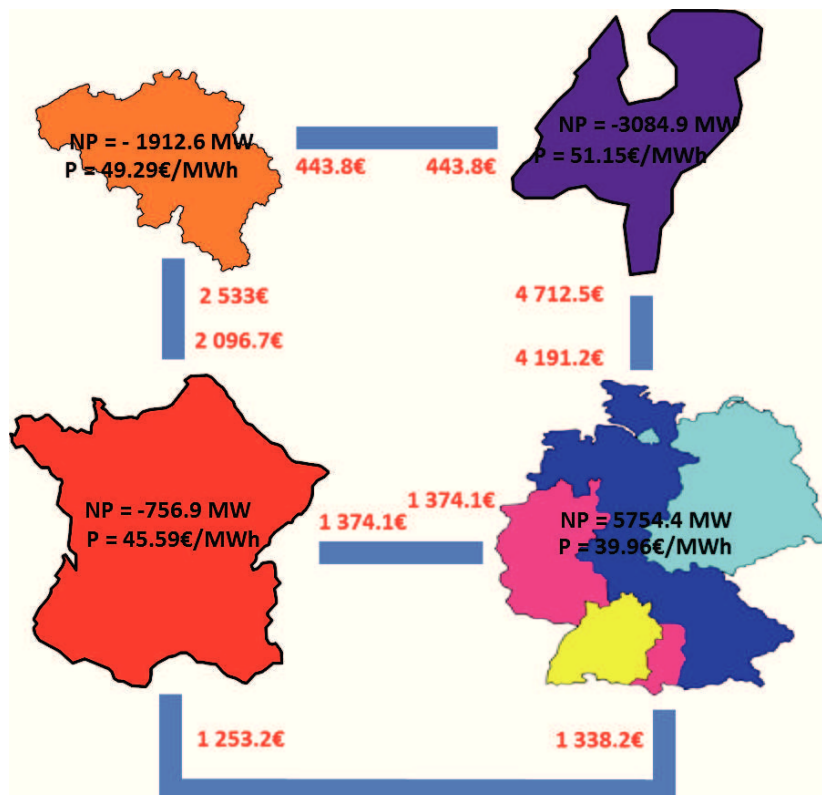


Figure 12: Resale cost per hub border on each side of the border

The hourly net income (income minus resale costs) should not lead to negative income per side of a hub border. In line with the resale methodology, the resale for that side of the hub border will initially be borne by its TSO. However, in case the income on a particular side of the hub border is

not sufficient to cover these resale costs, these costs will be borne pro rata by the other hub borders (shown in the iteration of the cycle in Figure 10). This is referred to as 'socialization'.

The examples shown above, with the congestion income and resale cost per hub border on each side of the border, are now combined in Figure 13; note that there is a necessity for socialization on the BE-FR border. Indeed, at both sides of the hub border, the resale cost exceeds the congestion income.

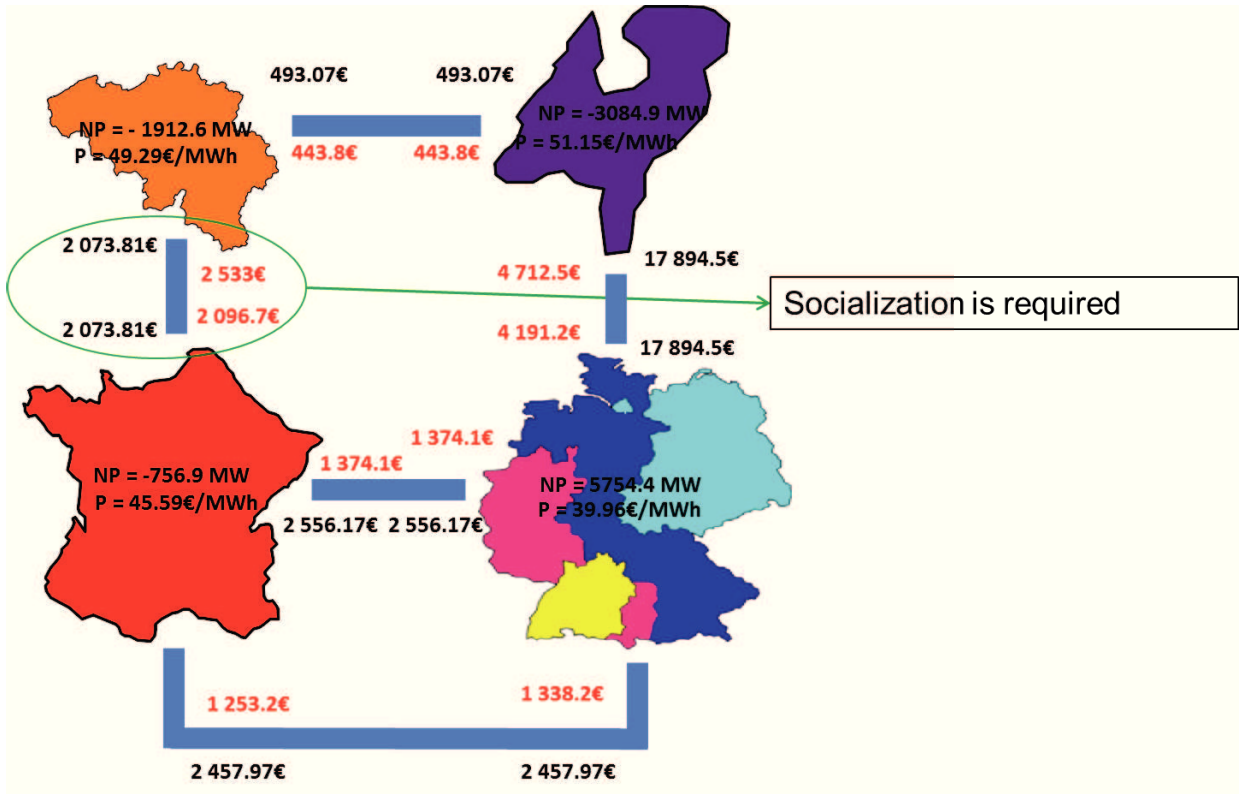


Figure 13: Combination of congestion income and resale costs on a hub border

After the socialization, the net income per side of the hub border is shown in Figure 14.

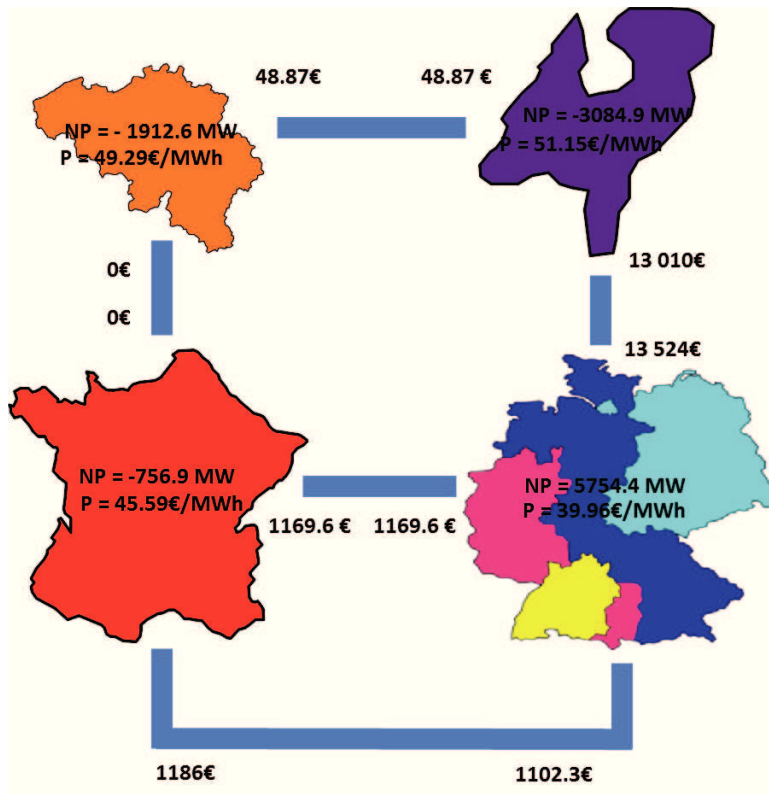


Figure 14: Net congestion income on a hub border

The remaining resale costs on the French - Belgian border has been shared on a pro rata basis among the other hub borders with a positive net congestion income. The share of each hub border, per side of the border, is shown in the table below.

Border	Additional resale cost due to socialization
DE-FR.DE	17.53€
DE-FR.FR	17.53€
DE-NL.DE	202.89€
DE-NL.NL	195.18€
BE-FR.BE	0€
BE-FR.FR	0€
BE-NL.BE	0.73€
BE-NL.NL	0.73€
DE-SZ	16.54€
FR-SZ	17.79€

Table 3: Additional resale costs, due to socialization, per hub border on each side of the border

8.5 Additional issue linked to the resale with Flow-Based daily allocation

In the previous chapters, we have already seen that there is a one-to-one relation between the Long Term ATC capacity and the available margins on day-ahead critical branches.

For the above mentioned reason, TSOs need to evaluate clearly what are the possible effects on the congestion income sharing, of the Long Term (non-harmonised) bilateral allocation of capacity on the one hand and of the fully coordinated Flow-Based allocation of capacity on the other.

Indeed, TSOs know that the Long Term allocation income will be received by the two TSOs issuing the capacity on that border. In line with the resale methodology, the resale will initially be borne by those TSOs. However, in case their income through the Flow-Based allocation is not sufficient to cover this, the resale costs for that border might be borne by other/all TSOs (socialization).

9 Sharing of the external pot

The net congestion on the internal and external borders is shown in Figure 15. For the external borders, the following method is applied:

- 50% of the net congestion income of DE-SZ border goes to the DE side of the open border
- 50% of the net congestion income of FR-SZ border goes to the FR side of the open border
- 50% of the net congestion income of DE-SZ and FR-SZ border goes to all borders (DE-FR, DE-NL, BE-FR, BE-NL, DE-SZ, FR-SZ) based on the additional aggregated flows on those borders.

This methodology of sharing the external pot is in line with the CBCPM philosophy and takes into account the "external" world. Both the sharing of the internal and external pot is based on AAFs. The sharing key for the external pot gives an incentive to the TSOs to provide the highest FB capacities to the DA market.

The results of the sharing key will be monitored during 2014 and 2015 (beginning of the year). The TSOs will provide a report by the end of 2015 to the NRAs containing an evaluation of the sharing key of the external pot with a recommendation either to remain to the chosen sharing key or to justify a new sharing key for the external pot in case unforeseen effects are identified.

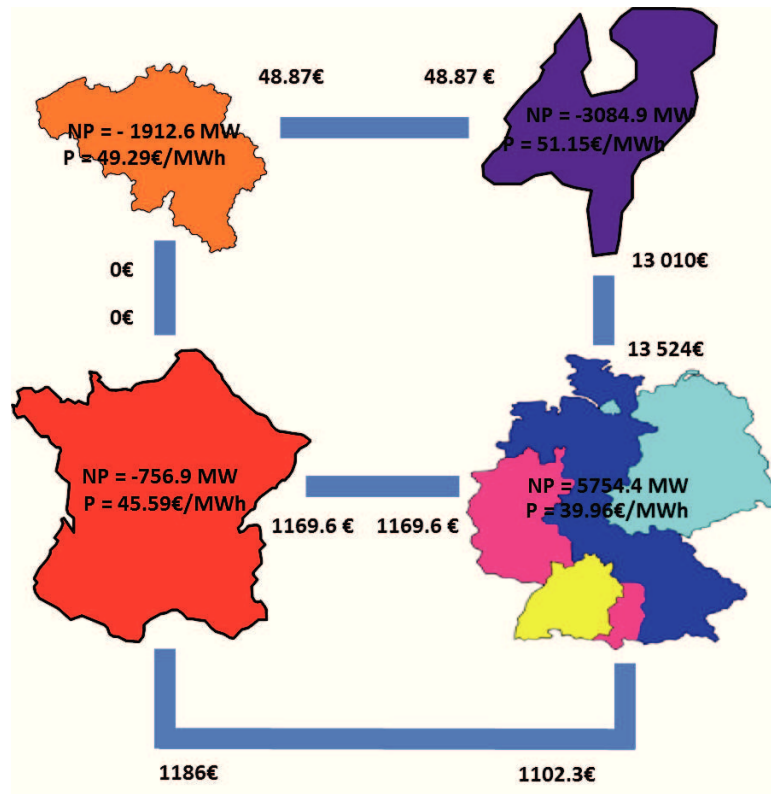


Figure 15: Net congestion income on a hub border

In Table 4, the net congestion per hub of the CWE region and for the slack zone is determined.

Hub	Net congestion income
BE	$0 + 48.87 = 48.87\text{€}$
DE	$13\,524 + 1\,169.6 + 1\,102.3/2 = 15\,244.80\text{€}$
FR	$1\,169.6 + 1\,186/2 = 1\,762.90\text{€}$
NL	$48.87 + 13\,010 = 13\,058.87\text{€}$
Slack zone	$(1\,186 + 1\,102.3)/2 = 1\,144.15\text{€}$

Table 4: The net congestion income for the hubs and the slack zone

The net congestion income of the slack zone is shared based on the AAFs on all borders (i.e. DE-FR, DE-NL, BE-FR, BE-NL, DE-SZ, FR-SZ). These flows are shown in Figure 16. The sum of the flows is 7667 MW. The results of the calculation of the shares are presented in Table 5.

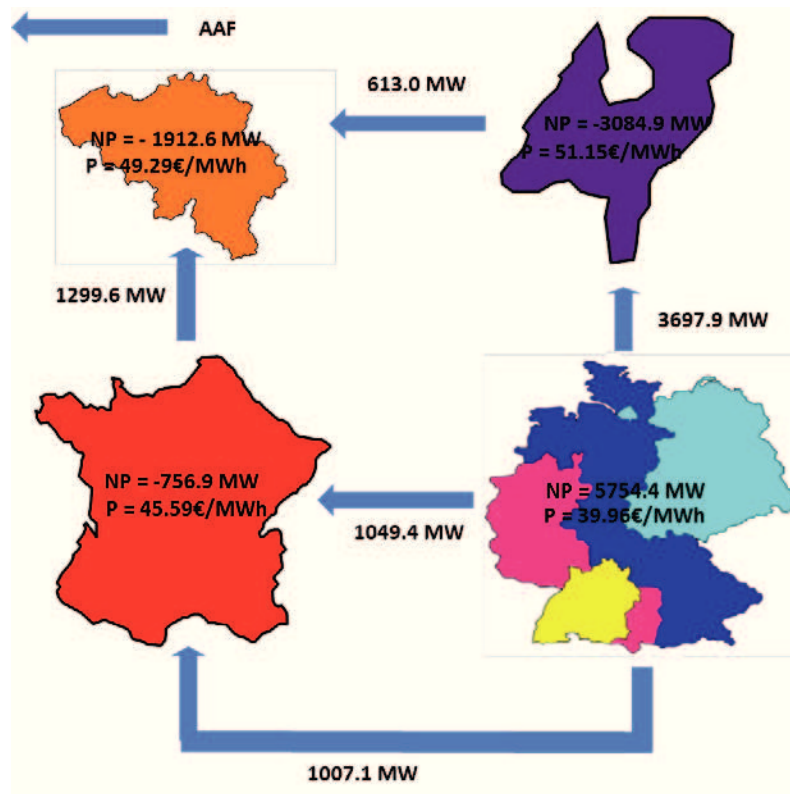


Figure 16: The additional aggregated flows

Border	Share
BE-FR	$1299.6/7667 = 16.95\%$
BE-NL	$613/7667 = 8.00\%$
DE-FR	$1049.4/7667 = 13.69\%$
DE-NL	$3697.9/7667 = 48.32\%$
Open borders	$1007.1/7667 = 13.14\%$

Table 5: The share for each hub border based on the AAFs

Based, on those shares, the net congestion income of the slack zone can be shared between all borders as shown in Table 6

Border	Share
BE-FR	16.95%*1144.15€ = 193.93€
BE-NL	8.00%*1144.15€ = 91.53€
DE-FR	13.69%*1144.15€ = 156.63€
DE-NL	48.32%*1144.15€ = 552.85€
Open borders	13.14%*1144.15€ = 150.34€

Table 6: The net congestion income of the slack zone shared between the hub borders

The final net congestion income for each hub is shown in Table 7.

Hub	Net congestion income
BE	$48.87 + 193.93/2 + 91.53/2 = 191.60\text{€}$
DE	$15\,244.80 + 156.63/2 + 552.85/2 + 150.34/2 = 15\,674.70$
FR	$1\,762.90 + 193.93/2 + 156.63/2 + 150.34/2 = 2\,013.35\text{€}$
NL	$13\,058.87 + 91.53/2 + 552.85/2 = 13\,381.10\text{€}$

Table 7: The final net congestion income for each hub

10 In case of extensions

10.1 Internal and external pot

The definition of the external pot implies that on the open borders, either “virtual AAFs” or “virtual prices” are defined for the computation. With the current FB configuration in CWE, both approaches lead to the same results. The easiest way to explain the approach in the current CWE area is by using “virtual AAFs” which is the difference between the net position (corrected from the Long Term Nominations), and the AAFs attributed to each internal border. In CWE, there are 2 open borders where these “virtual AAFs” are required: one for the French hub and one for the German hub. All the flows exiting CWE through one of the two open borders re-enter CWE through the other open border. As such, these “virtual AAFs” are directly known on the external borders, and are indeed real “AAFs” (no additional rule is required to compute them). In case of extension however, additional assumptions will be necessary to define the “virtual AAFs” (to be used with the prices set by the FB MC). Another approach is possible by considering a slack zone closing the system (using real “AAFs” without additional rules), for which a virtual price needs to be given. The slack zone merges all the non-CWE hubs. With more than three open borders, it becomes impossible to define uniquely the AAFs between two open borders without additional rules. CWE TSOs have not decided yet on the precise rules to apply in case of extension, but agreed on the fact that the minimization of the external pot would be the primary target when defining these virtual AAFs or virtual price.

10.1.1 Determination of the unique price of the slack zone

In Figure 17, the CWE region under a possible extension is presented. In this example, there are 4 open borders and a virtual price of the slack zone has to be determined.

The minimum of the external pot is achieved for the slack zone having a price in between 57€ and 59€. Between those prices, the external pot is equal. In order to have a congestion income on each border, the average price between the two prices can be taken for example: i.e. 58€.

- Border DE- SZ is valued at 1 k€
 - Border Z- SZ is valued at 1 k€
 - Border Y- SZ is valued at 0,5 k€
 - Border FR- SZ is valued at 18 k€
- ⇒ Algebraic Sum = 20,5 k€
 ⇒ Sum of absolute values = 20,5 k€

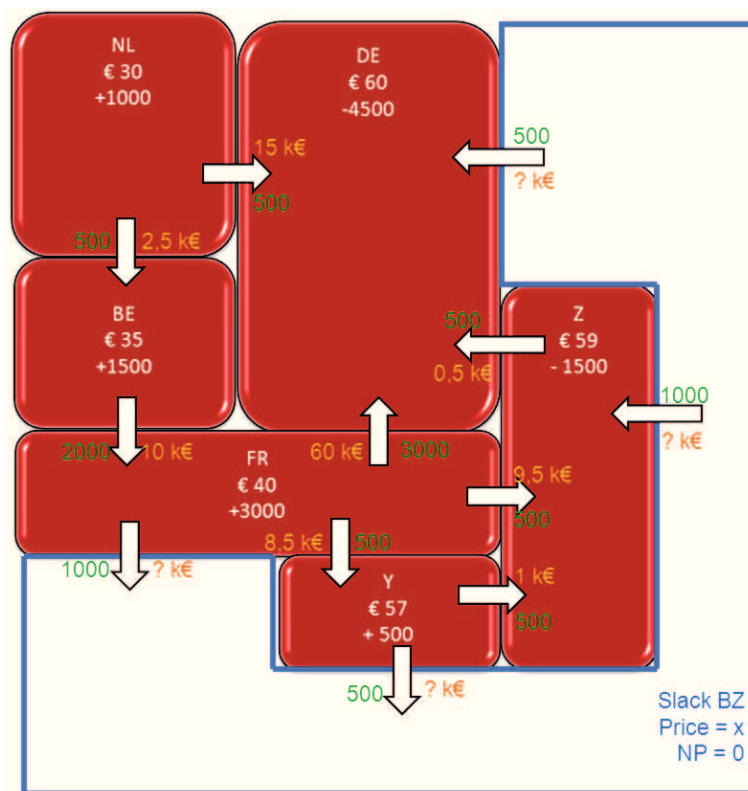


Figure 17: A market situation with 4 open borders

A similar method is applicable in the case of non-intuitive flows.

10.1.2 Determination of the "virtual" AAFs

In Figure 18, the CWE region under a possible extension is presented. In this example, there are 4 open borders and the "virtual" AAFs have to be determined.

Even by minimizing the external pot, there is no unique solution for the "virtual" AAFs. Therefore, additional assumptions are needed to come to a unique selection of the flows between the hubs with open borders.

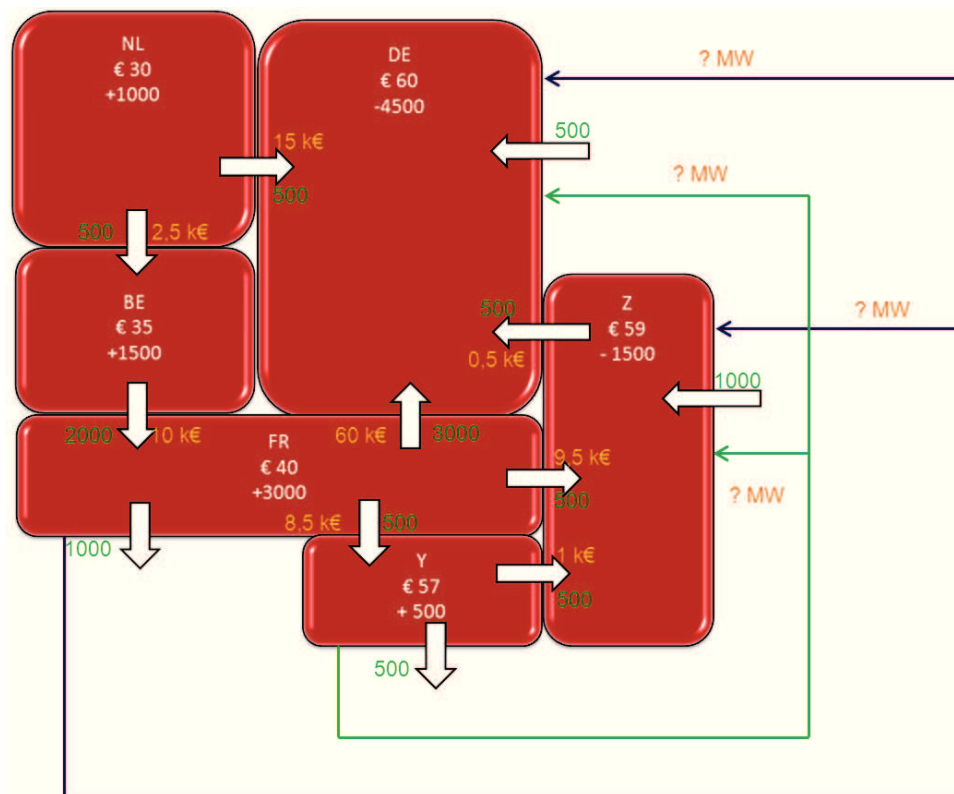


Figure 18: A market situation with 4 open borders

10.2 Resale methodology in case of extension

The CWE resale methodology is applicable in case of extensions.

This is illustrated with the help of an example. There is a resale of 1000MW from France to Germany as shown in Figure 19. Since the price difference is 20€, the resale cost on that border is 20 000€. Looking at the French side of the border, 850MW of 1000MW is on the internal borders thus 85% of the half of the resale cost is awarded to the French side of the border, i.e. 8500€. The remaining part is attributed to the border between France and the slack zone. The same method is applicable at the German side of the border. 750MW of 1000MW is on the internal borders, giving a resale cost of 7500€ at the German side of the border. The remaining part, i.e. 2500€ is awarded to the border between Germany and the slack zone.

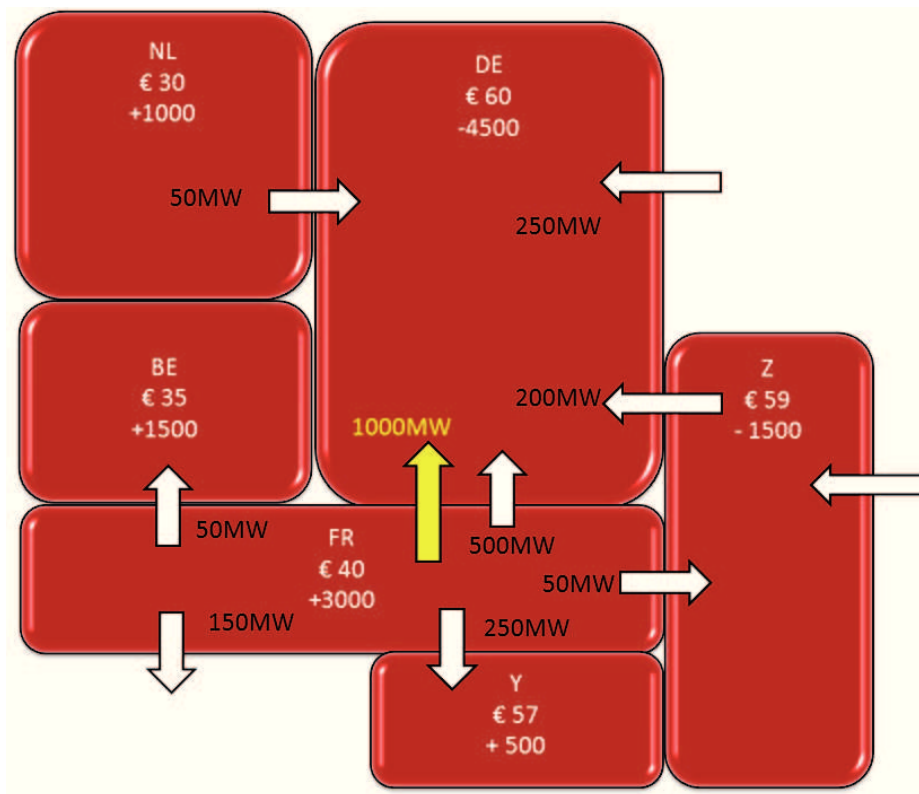


Figure 19: Resales under FB MC extension

10.3 Treatment of external pot in case of extensions

In case of extension, the following method is applied:

- 50% of the net congestion income of each open border goes to the corresponding hub of the FB region (i.e. the hub in the FB region having the open border).
- 50% of the net congestion income of each open border goes to all borders (i.e. closed and open borders) based on the additional aggregated flows on all open and closed borders.

11 Glossary

AAFs	Additional aggregated flow
ATC	Available Transfer Capacity
ATC MC	ATC Market Coupling
CB	Critical Branch
CBCPM	Cross Border Clearing Price x Market flows
CI	Congestion Income (from day-ahead Market Coupling)
CIA WG	Congestion Income Allocation working group (TSO only)
CP	Clearing Price
CWE	Central Western Europe
D-1	Day Ahead
DA	Day Ahead
DAMW	Day Ahead Market Welfare
FB	Flow-Based
FBMC	Flow-Based Market Coupling
LT	Long Term
LTA	Allocated Long Term Transmission Capacity
LTN	Nominated Long Term Transmission Capacity
MC	Market Coupling
NP	Net Position (sum of commercial exchanges for one bidding area)
NTC	Net Transmission Capacity
PTDF	Power Transfer Distribution Factor
RAM	Remaining Available Margin
SP	Shadow Price
SZ	Slack Zone
TSO	Transmission System Operator
UIOSI	Use It Or Sell It

Annex 1 Numerical example and proofs of resale costs versus flow-based income

1.1 Example: Resale costs higher than hourly congestion income in Flow-Based.

In order to understand better how the resale costs 'work' in Flow-Based, let's assume the following example, for illustration purpose:

- Critical Branch CB1: internal line with increasing flows for any export outside hub A - margin available 100MW
- Resale of capacity from Hub A towards Hub B: 200MW – influencing factor on CB1 = 20%
- Resale of capacity from Hub A towards Hub C: 250MW – influencing factor on CB1 = 30%
- The double export of energy from Hub A is unrealistic since there is not enough production in Market A for this configuration.

In this situation, we know that we have sold too much capacity simultaneously, on both interconnections, however there is no physical risk due to the constraint on the production availability in hub A.

Nevertheless, if the clearing result of Market Coupling leads to the congestion of the Critical Branch CB1, we will have the following situation (by assuming a shadow price on CB1 = 50€):

- Overall congestion income :
Margin on CB1 × Shadow Price on CB1 = 100 × 50 = **5 000€**
- Resale cost linked to 200MW of capacity between Hub A and Hub B
(Capacity resold × influencing factor CB1)⁵ × Shadow Price CB1⁶ = 200 × 20% × 50 = **2 000€**
- Resale cost linked to 250MW of capacity between Hub A and Hub C
(Capacity resold × influencing factor CB1 × Shadow Price CB1⁷ = 250 × 30% × 50 = **3 750€**

In this situation, we have a resale cost that is higher than the total hourly congestion income from the Flow-Based Market coupling. In addition, we have to point out the fact that the congestion of this Critical Branch might appear even if the market results is not a double export from Hub A.

1.2 Example (intuitive) for the resale proof

The example described in this section shows that the resale cost are covered by the hourly congestion income as long as the LTA domain is within FB domain. The three nodes (shown in Figure 20) are connected by three lines that have equal impedance. Node C acts as the swingbus / slacknode. Let's assume that the lines are unloaded and have a maximum capacity of 9MW.

⁵ Margin freed by the resale of capacity on the critical branch

⁶ Calculation linked to the high Level Property of Flow-Based allocation. In that respect, the Price in market A will be 2 000/200 = 10€ less expensive than in Market B.

⁷ Calculation linked to the high Level Property of Flow-Based allocation. In that respect, the Price in market A will be 3 750/250 = 15€ less expensive than in Market C.

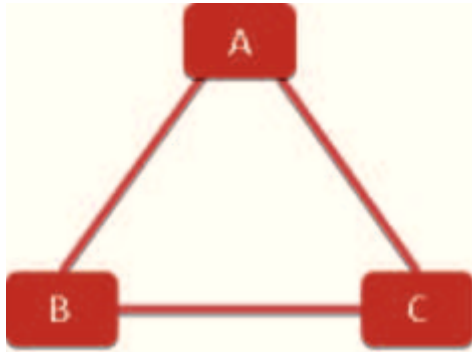


Figure 20: Example with three nodes

$$\begin{matrix}
 AB: & \begin{bmatrix} 1/3 & -1/3 \end{bmatrix} \\
 BC: & \begin{bmatrix} 1/3 & 2/3 \end{bmatrix} \\
 AC: & \begin{bmatrix} 2/3 & 1/3 \end{bmatrix} \\
 AB: & \begin{bmatrix} -1/3 & 1/3 \end{bmatrix} \\
 BC: & \begin{bmatrix} -1/3 & -2/3 \end{bmatrix} \\
 AC: & \begin{bmatrix} -2/3 & -1/3 \end{bmatrix}
 \end{matrix}
 \begin{matrix}
 [NP(A)] \\
 [NP(B)]
 \end{matrix}
 \leq
 \begin{matrix}
 9 \\
 9 \\
 9 \\
 9 \\
 9 \\
 9
 \end{matrix}$$

Figure 21: PTDF matrix

The FB domain is visualized in Figure 22.

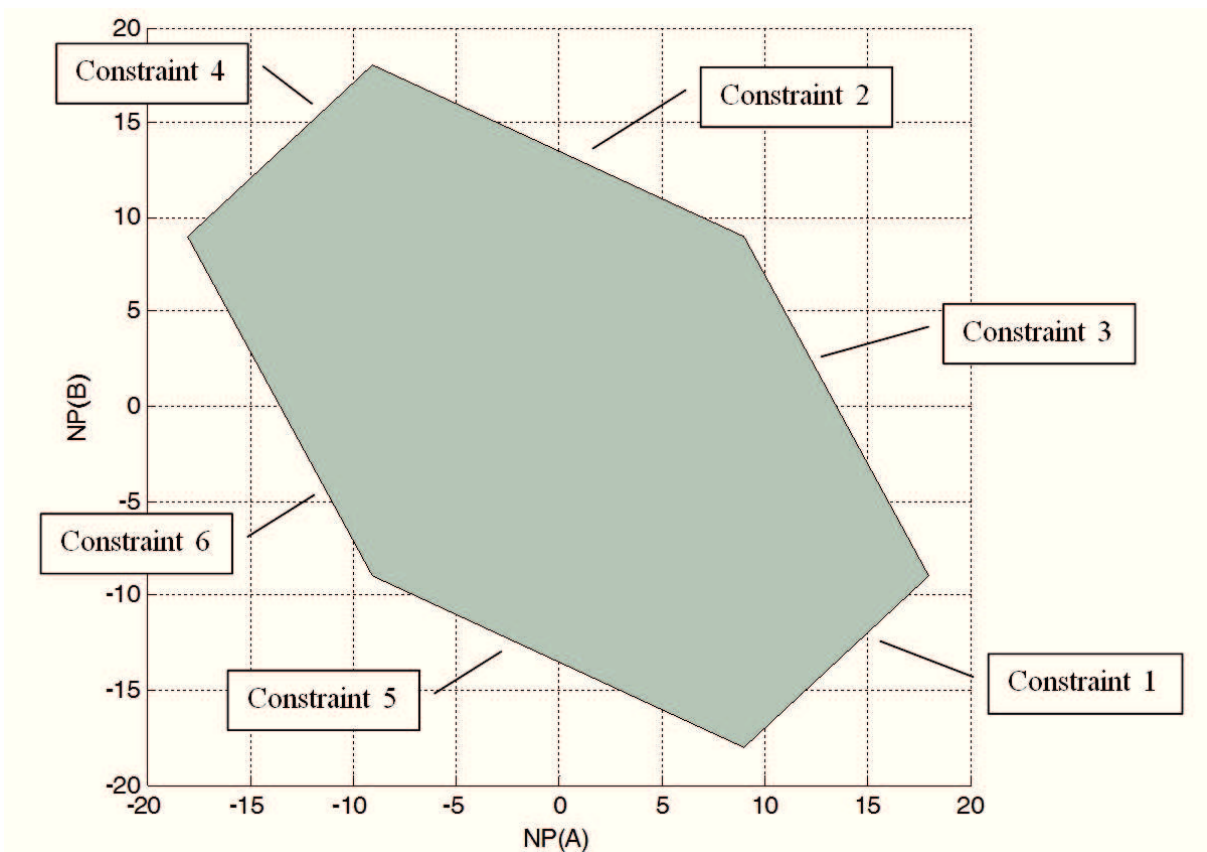


Figure 22: FB domain

The LTA are as follows:

$$\begin{matrix}
 A > B \\
 A > C \\
 B > C \\
 B > A \\
 C > A \\
 C > B
 \end{matrix}
 =
 \begin{bmatrix}
 13.5 \\
 0 \\
 13.5 \\
 0 \\
 13.5 \\
 0
 \end{bmatrix}$$

The LTA domain is shown, together with the FB one, in the following figure.

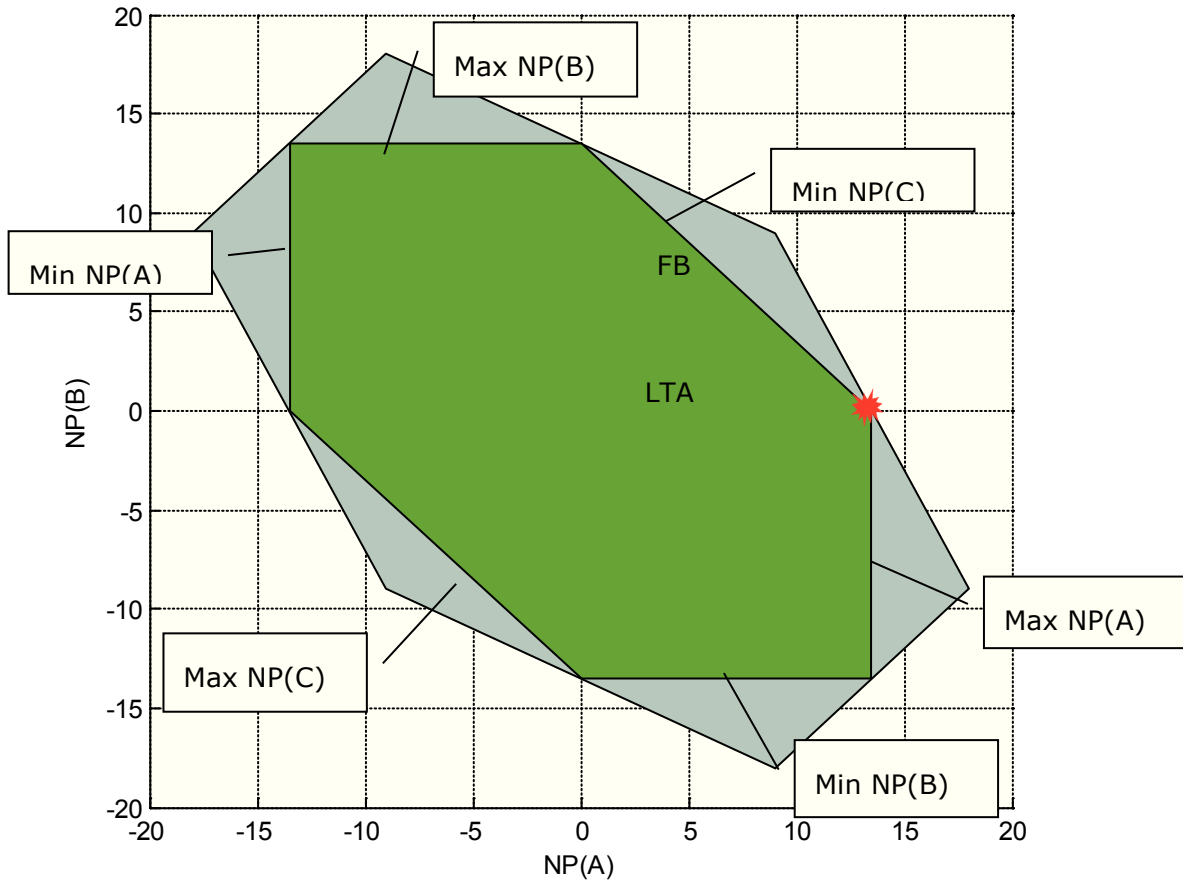


Figure 23: FB and LT domain

After the FBMC, a congested situation appears. Constraint 3 is hit (★), and the following shadow price results: $\mu = 30 \text{ €}$.

The resulting prices and net positions are:

$$P_A = 10 \text{ €}, NP_A = 13.5$$

$$P_B = 20 \text{ €}, NP_B = 0$$

$$P_C = 30 \text{ €}, NP_C = -13.5$$

Maximum Resale Costs compensated at price spread is "Max RC":

$$\text{Max RC} = \sum_i \sum_{j \neq i} LTA_{i \rightarrow j} \cdot \max((P_j - P_i), 0) = 13.5 * 10 + 13.5 * 10 + 0 = 270 \text{ €}$$

For each border $i \rightarrow j$, a set of bilateral exchanges $BE_{i \rightarrow j}$ is:

$$\begin{cases} BE_{i \rightarrow j} = LTA_{i \rightarrow j} & \text{if } P_j > P_i \\ BE_{i \rightarrow j} = -LTA_{j \rightarrow i} & \text{if } P_j < P_i \\ BE_{i \rightarrow j} = 0 & \text{if } P_j = P_i \end{cases}$$

$$BE_{A \rightarrow B} = 13.5, BE_{B \rightarrow A} = -13.5$$

$$BE_{A \rightarrow C} = 0, BE_{C \rightarrow A} = 0$$

$$BE_{B \rightarrow C} = 13.5, BE_{C \rightarrow B} = -13.5$$

Consider Q'_i as the net position associated with this set of exchanges $BE_{i \rightarrow j}$:

$$\forall i \quad Q'_i = \sum_{j \neq i} BE_{i \rightarrow j} \quad [b]$$

$$\forall i, j \quad BE_{i \rightarrow j} = -BE_{j \rightarrow i}$$

$$\sum_i Q'_i = \sum_i \sum_{j \neq i} BE_{i \rightarrow j} = 0 \quad [c]$$

$$Q'_A = BE_{A \rightarrow B} + BE_{A \rightarrow C} = 13.5$$

$$Q'_B = BE_{B \rightarrow A} + BE_{B \rightarrow C} = -13.5 + 13.5 = 0$$

$$Q'_C = BE_{C \rightarrow A} + BE_{C \rightarrow B} = 0 - 13.5 = -13.5$$

$$\text{Indeed, } \sum_i Q'_i = 0.$$

With [a] and [b], we are now able to rewrite:

$$\text{Max RC} = \sum_i \sum_{j > i} BE_{i \rightarrow j} \cdot (P_j - P_i) = - \sum_i (Q'_i \cdot P_i) \quad [d]$$

$$\text{Max RC} = BE_{A \rightarrow B} \cdot (P_B - P_A) + BE_{A \rightarrow C} \cdot (P_C - P_A) + BE_{B \rightarrow C} \cdot (P_C - P_B) = -P_A \cdot (BE_{A \rightarrow B} + BE_{A \rightarrow C}) - P_B \cdot (-BE_{A \rightarrow B} + BE_{B \rightarrow C}) - P_C \cdot (-BE_{A \rightarrow C} - BE_{B \rightarrow C}) = -P_A Q'_A - P_B Q'_B - P_C Q'_C = -10 \cdot 13.5 - 20 \cdot 0 - 30 \cdot (-13.5) = 270 \text{ €}$$

Moreover the net position Q'_i is within the FB domain. Then:

$$\forall l \in \text{CB}, \sum_i Q'_i \cdot \text{PTDF}_{i,l} \leq m_l \quad [e]$$

Where CB is the group of all critical branches and m_l is the margin (available for DA MC) on the critical branch l. This margin is positive if the LT domain is included in the FB domain.

Indeed, the net positions are within the FB domain:

$$\begin{array}{l} \text{AB:} \\ \text{BC:} \\ \text{AC:} \\ \text{AB:} \\ \text{BC:} \\ \text{AC:} \end{array} \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 2/3 \\ 2/3 & 1/3 \\ -1/3 & 1/3 \\ -1/3 & -2/3 \\ -2/3 & -1/3 \end{bmatrix} \begin{bmatrix} 13.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 4.5 \\ 9 \\ -4.5 \\ -4.5 \\ -9 \end{bmatrix} \leq \begin{bmatrix} 9 \\ 9 \\ 9 \\ 9 \\ 9 \\ 9 \end{bmatrix}$$

The Congestion Income « CI » collected in D-1 can be written as :

$$\text{CI} = - \sum_i (Q_i \cdot P_i) = \sum_{l \in \text{CB}} (\mu_l \cdot m_l) \quad [f]$$

where μ_l is the shadow price of the critical branch l.

The Congestion Income in our example amounts

based on the computation with net positions and prices:

$$CI = -10 * 13.5 - 20 * 0 - 30 * -13.5 = 270 \text{ €}$$

based on the computation with shadow price and margin:

$$CI = 9 * 30 = 270 \text{ €}$$

Flow-Based clearing also has the following properties⁸ :

$$\forall l \in CB, \mu_l \geq 0 \tag{g}$$

$$\exists P_{ref} \text{ such that } \forall i, P_i = P_{ref} - \sum_{l \in CB} PTDF_{i,l} \cdot \mu_l \tag{h}$$

With [f] and [d], we finally have:

$$CI - \text{Max RC} = \sum_{l \in CB} \mu_l \cdot m_l - \left(- \sum_i Q'_i \cdot P_i \right)$$

With [h],

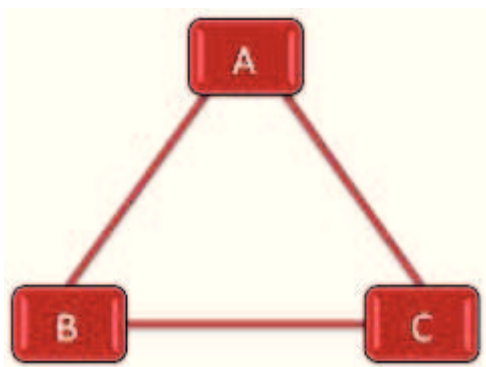
$$\begin{aligned} &= \sum_{l \in CB} \mu_l \cdot m_l + \sum_i Q'_i \cdot (P_{ref} - \sum_{l \in CB} PTDF_{i,l} \cdot \mu_l) \\ &= \sum_{l \in CB} \mu_l \cdot m_l + P_{ref} \cdot \sum_i Q'_i - \sum_i (Q'_i \cdot \sum_{l \in CB} PTDF_{i,l} \cdot \mu_l) \end{aligned}$$

With [c],

$$= \sum_{l \in CB} \mu_l (m_l - \sum_i Q'_i \cdot PTDF_{i,l})$$

1.3 Example (non-intuitive) for the resale proof

The example described in this section shows that the resale cost are covered by the hourly congestion income as long as the LTA domain is within the FB domain. The three nodes are connected by three lines that have equal impedance as shown in Figure 24. Node C acts as the swingbus / slacknode. Let's assume that the lines are unloaded and have different maximum capacities.



$$\begin{matrix} AB: & \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 2/3 \end{bmatrix} \\ BC: & \begin{bmatrix} 2/3 & 1/3 \\ -1/3 & 1/3 \end{bmatrix} \\ AC: & \begin{bmatrix} -1/3 & 1/3 \\ -1/3 & -2/3 \end{bmatrix} \end{matrix} \begin{bmatrix} NP(A) \\ NP(B) \end{bmatrix} \leq \begin{bmatrix} 14.67 \\ 9.67 \\ 15.33 \\ 3.33 \\ 8.33 \\ 2.67 \end{bmatrix}$$

⁸ Based on the following FB equation: $\frac{P_j - P_i}{PTDF_i - PTDF_j} = \mu_l \geq 0$

Figure 24: Example with three nodes

Figure 25: PTFD matrix

The FB domain is visualized in the graph hereunder.

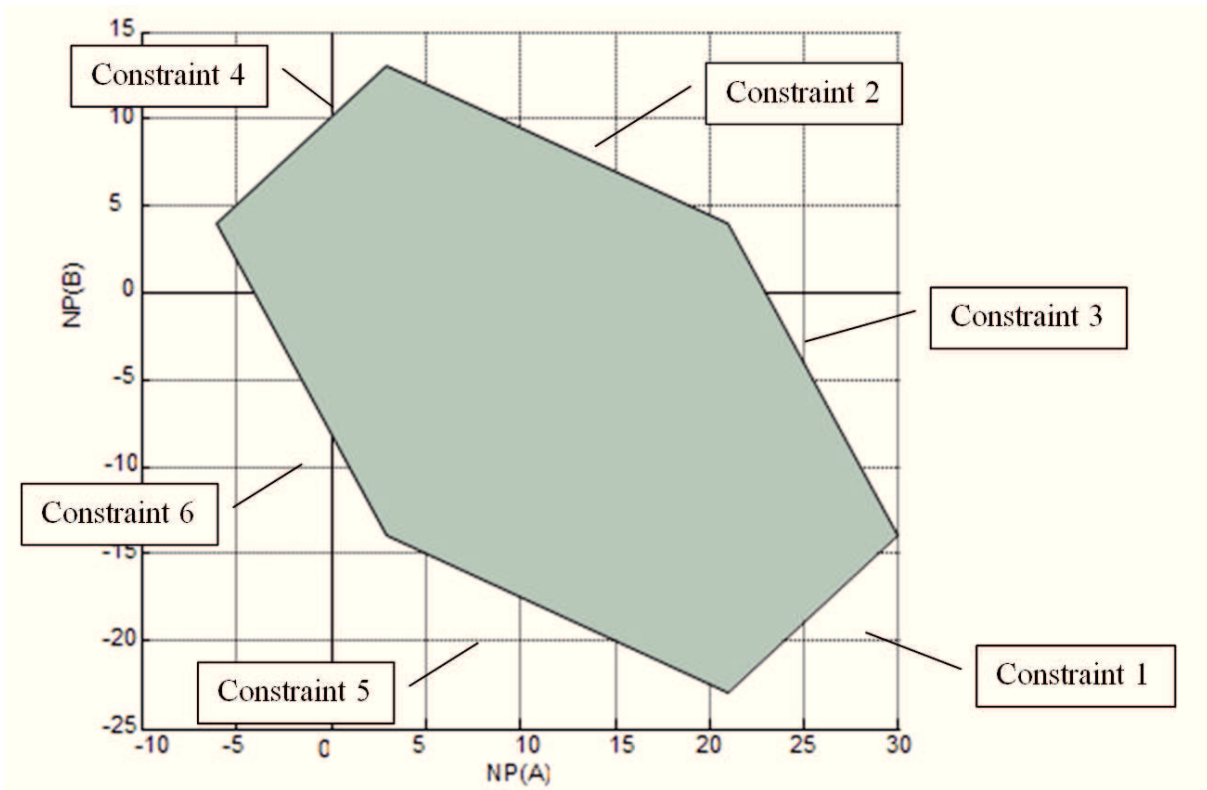


Figure 26: FB domain

The LTA are as follows:

$$\begin{bmatrix} A > B \\ A > C \\ B > C \\ B > A \\ C > A \\ C > B \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 10 \\ 0 \\ 0 \\ 8 \end{bmatrix}.$$

The LTA domain is shown, together with the FB one, in the following figure.

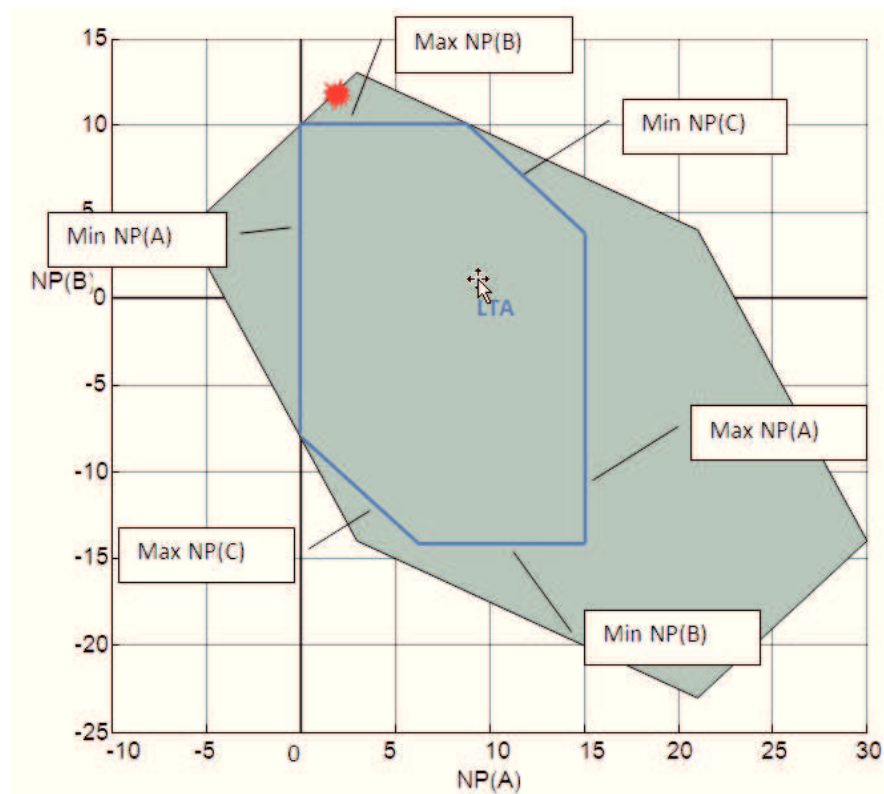


Figure 27: FB and LTA domain

After the FBMC, a congested non-intuitive situation appears. Constraint 4 is hit (★), and the following shadow price results: $\mu = 30 \text{ €}$.

The resulting prices and net positions are:

$$P_A = 0 \text{ €}, \quad NP_A = 2$$

$$P_B = -20 \text{ €}, \quad NP_B = 12$$

$$P_C = -10 \text{ €}, \quad NP_C = -14$$

Maximum Resale Costs compensated at price spread is « Max RC » :

$$\text{Max RC} = \sum_i \sum_{j \neq i} LTA_{i \rightarrow j} \cdot \max((P_j - P_i), 0) = 0 + 0 + 10 * (-10 + 20) + 0 = 100 \text{ €}$$

For each border $i \rightarrow j$, a set of bilateral exchanges $BE_{i \rightarrow j}$ is:

$$\begin{cases} BE_{i \rightarrow j} = LTA_{i \rightarrow j} & \text{if } P_j > P_i \\ BE_{i \rightarrow j} = -LTA_{j \rightarrow i} & \text{if } P_j < P_i \\ BE_{i \rightarrow j} = 0 & \text{if } P_j = P_i \end{cases}$$

$$BE_{A \rightarrow B} = 0, \quad BE_{B \rightarrow A} = 0$$

$$BE_{A \rightarrow C} = 0, \quad BE_{C \rightarrow A} = 0$$

$$BE_{B \rightarrow C} = 10, \quad BE_{C \rightarrow B} = -10$$

Consider Q'_i as the net position associated with this set of exchanges $BE_{i \rightarrow j}$:

$$\forall i \quad Q'_i = \sum_{j \neq i} BE_{i \rightarrow j} \quad [b]$$

$$\forall i, j \quad BE_{i \rightarrow j} = -BE_{j \rightarrow i}$$

$$\sum_i Q'_i = \sum_i \sum_{j \neq i} BE_{i \rightarrow j} = 0 \quad [c]$$

$$Q'_A = BE_{A \rightarrow B} + BE_{A \rightarrow C} = 0 + 0 = 0$$

$$Q'_B = BE_{B \rightarrow A} + BE_{B \rightarrow C} = 0 + 10 = 10$$

$$Q'_C = BE_{C \rightarrow A} + BE_{C \rightarrow B} = 0 - 10 = -10$$

$$\text{Indeed, } \sum_i Q'_i = 0.$$

With [a] and [b], we are now able to rewrite:

$$\text{Max RC} = \sum_i \sum_{j > i} BE_{i \rightarrow j} \cdot (P_j - P_i) = - \sum_i (Q'_i \cdot P_i) \quad [d]$$

$$\begin{aligned} \text{Max RC} &= BE_{A \rightarrow B} \cdot (P_B - P_A) + BE_{A \rightarrow C} \cdot (P_C - P_A) + BE_{B \rightarrow C} \cdot (P_C - P_B) = -P_A \cdot (BE_{A \rightarrow B} - BE_{A \rightarrow C}) - P_B \cdot (BE_{A \rightarrow B} - \\ &BE_{B \rightarrow C}) - P_C \cdot (BE_{A \rightarrow C} - BE_{B \rightarrow C}) = -P_A Q'_A - P_B Q'_B - P_C Q'_C = 0 \cdot 0 - (-20 \cdot 10) - (-10 \cdot -10) = 200 - 100 = 100 \\ &\text{€} \end{aligned}$$

Moreover the net position Q'_i is within the FB domain. Then:

$$\forall l \in \text{CB}, \sum_i Q'_i \cdot \text{PTDF}_{i,l} \leq m_l \quad [e]$$

where CB is the group of all critical branches and m_l is the margin (available for DA MC) on the critical branch l. This margin is positive if the LT domain is included in the FB domain.

Indeed, the net positions are within the FB domain:

$$\begin{array}{l} \text{AB:} \\ \text{BC:} \\ \text{AC:} \\ \text{AB:} \\ \text{BC:} \\ \text{AC:} \end{array} \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 2/3 \\ 2/3 & 1/3 \\ -1/3 & 1/3 \\ -1/3 & -2/3 \\ -2/3 & -1/3 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \end{bmatrix} = \begin{bmatrix} -3.33 \\ 6.67 \\ 3.33 \\ 3.33 \\ -6.67 \\ -3.33 \end{bmatrix} \leq \begin{bmatrix} 14.67 \\ 9.67 \\ 15.33 \\ 3.33 \\ 8.33 \\ 2.67 \end{bmatrix}$$

The Congestion Income « CI » collected in D-1 can be written as :

$$\text{CI} = - \sum_i (Q_i \cdot P_i) = \sum_{l \in \text{CB}} (\mu_l \cdot m_l) \quad [f]$$

where μ_l is the shadow price of the critical branch l.

The Congestion Income in our example amounts

based on the computation with net positions and prices:

$$CI = -0 * 2 - (-20 * 12) - (-10 * -14) = 240 - 140 = 100 \text{ €}$$

based on the computation with shadow price and margin:

$$CI = 3.33 * 30 = 100 \text{ €}$$

Flow-Based clearing also has the following properties⁹ :

$$\forall l \in CB, \mu_l \geq 0 \quad [g]$$

$$\exists P_{ref} \text{ such that } \forall i, P_i = P_{ref} - \sum_{l \in CB} PTDF_{i,l} \cdot \mu_l \quad [h]$$

With [f] and [d], we finally have:

$$CI - \text{Max RC} = \sum_{l \in CB} \mu_l \cdot m_l - (-\sum_i Q'_i \cdot P_i)$$

$$\text{With [h]} \quad = \sum_{l \in CB} \mu_l \cdot m_l + \sum_i Q'_i \cdot (P_{ref} - \sum_{l \in CB} PTDF_{i,l} \cdot \mu_l)$$

$$= \sum_{l \in CB} \mu_l \cdot m_l + P_{ref} \cdot \sum_i Q'_i - \sum_i (Q'_i \cdot \sum_{l \in CB} PTDF_{i,l} \cdot \mu_l)$$

$$\text{With [c],} \quad = \sum_{l \in CB} \mu_l (m_l - \sum_i Q'_i \cdot PTDF_{i,l})$$

$$\text{With [g] and [e],} \quad \geq 0$$

In our example, the Congestion Income is equal to the Resale Costs:

$$CI - \text{Max RC} = 100 - 100 = 0$$

⁹ Based on the following FB equation: $\frac{P_j - P_i}{PTDF_i - PTDF_j} = \mu_l \geq 0$