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Annex 16.18 Flow-Based "intuitive" explained

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Glossary

ATC	Available Transfer Capacity
B&B	Branch and Bound
CB	Critical Branch
CWE	Central Western Europe (Belgium, France, Germany, Luxembourg, Netherlands)
DA	Day Ahead
DAM	Day Ahead Market
DAMW	Day-Ahead Market Welfare
FB	Flow Based
FBI	Flow Based Intuitive
MIC	Minimum Income Condition (order type of the Iberian market)
NP or NEX	Net Position or Net Export Position (sum of commercial exchanges for one bidding area)
PCR	Price Coupling of Regions
PTDF	Power Transfer Distribution Factor
PX	Power Exchange
RAM	Remaining Available Margin
TSO	Transmission System Operator

1 Context

Within the CWE FB project one of the FB market coupling options has always been “intuitive” FB: the non-intuitive exchanges that could possibly result from a market coupling under FB network constraints are being suppressed by the algorithm. Much information on this subject has been published via the feasibility report, the intuitiveness report and the different market forums[1] as well as via the public Euphemia documentation[2] of the PCR PXs.

Yet this information has been perceived as too scattered, and a proper explanation of how “intuitive” FB works is hard to obtain with so many sources to consider. This document compiles an overview of the information on “intuitiveness” from these different sources and provides explicit references to the other documents where this is more appropriate. The focus on this document will be on functionality, rather than motivation of the choice for “intuitiveness”.

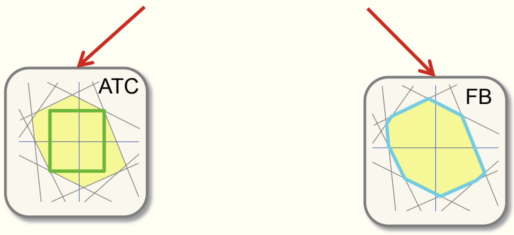
[1] See http://www.casc.eu/en/Resource-center/CWE-Flow-Based-MC/Documentation

[2] Available from all PCR PXs websites, e.g. http://www.apxgroup.com/wp-content/uploads/Euphemia-public-description-Nov-20131.pdf

2 Flow based market coupling

Market coupling under FB differs from ATC only as far as network constraints are considered. Otherwise the same (type of) market orders can be submitted, and the algorithm is faced with the challenge of finding solutions that respect all network constraints, yet maximize DAM welfare:

- All the bids of the local/national Power exchanges are brought together in order to be matched by a centralized algorithm.
- Objective function: Maximize Day-ahead Market Welfare
- Control variables: Net positions
- Subject to: ∑ net positions = 0
  Grid constraints



Under ATC it can easily be demonstrated that all resulting exchanges must be "intuitive", they must be scheduled from low to high prices. The reasoning is simple:

Imagine an exchange from market A to market B, where market A has a higher price than market B. Since the A to B exchange has no impact on other exchanges, a solution with more welfare exists, by reducing A to B. Therefore such a solution cannot be optimal, and by contradiction we prove that ATC solutions must be intuitive.

Under FB an exchange A to B does influence other exchanges. Consider Figure 1 which illustrates a FB domain and the red dot illustrates a clearing point (or market coupling solution). The red arrows indicate directions the solution cannot move to (outside the domain); the green arrows indicate direction the solution can move to (inside the domain).

We learn that B to C cannot increase, but can decrease. If B to C was non-intuitive, it would have been possible to decrease the exchange and increase welfare. If the solution is optimal, it must be that B to C is intuitive.

We learn that A to B cannot decrease, but can increase. If A to B would be non-intuitive, it will stay so, since it cannot be decreased. If it was intuitive, a more optimal solution would exists by increasing the exchange. If the solution is optimal, it must be that A to B is non-intuitive.

The reason that A to B is non-intuitive is because it frees up some capacity on a constraining CB. This freed capacity is then used to exchange more between B and C. This suggests that the loss in welfare on A to B is offset by the gain in welfare due to the additional B to C exchanges.

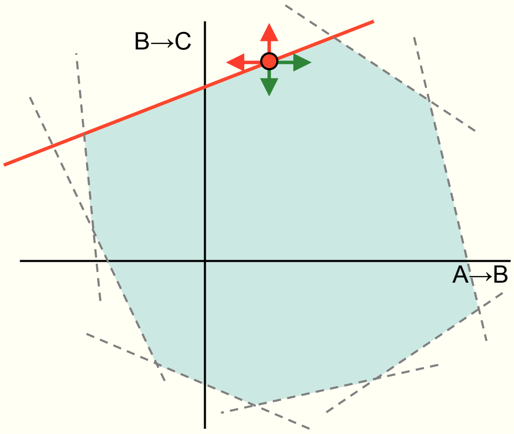


Figure 1 Illustration of FB domain and non-intuitive solution (red dot).

Conclusion

- Under ATC any exchange is guaranteed to be scheduled from low to high price;
- Under FB no a-priori statements can be made on the intuitiveness of solutions;
- Non-intuitive exchanges relieve congested CBs, and allow more beneficial trades to use the relieved CB;

3 Enforcing intuitiveness

Under FB it is possible to end up with non-intuitive solutions. We now consider a "patch" to suppress these non-intuitive solutions. To illustrate what we expect the patch to do, consider the welfare plots in Figure 2.

The example is a 3 market example, which can be plotted on 2D plane indicating the net positions of markets 1 and 2. The net position of market 3 follows from the balance constraint: $nex_3 = -nex_1 - nex_2$.

Figure A

We illustrate the different net positions of markets 1 and 2 and plot the corresponding DAM welfare on the z-axis. The welfare plot corresponds with the underlying order books of the three markets.

The welfare plot has a clearly defined optimum, which corresponds with the exchanges that would result in case no network restrictions applied (the infinite capacity case). The isolated solution (no exchanges) is illustrated too.

Figure B

Parts of both the ATC (white dotted line) and FB (black dotted line) domains are illustrated. The black curved lines are ISO welfare lines (i.e. lines where the welfare is constant). Since the FB domain in our example is larger than the ATC domain, it is possible to realize more welfare under FB than under ATC, corresponding to an ISO welfare line closer to the unconstrained solution.

Figure C

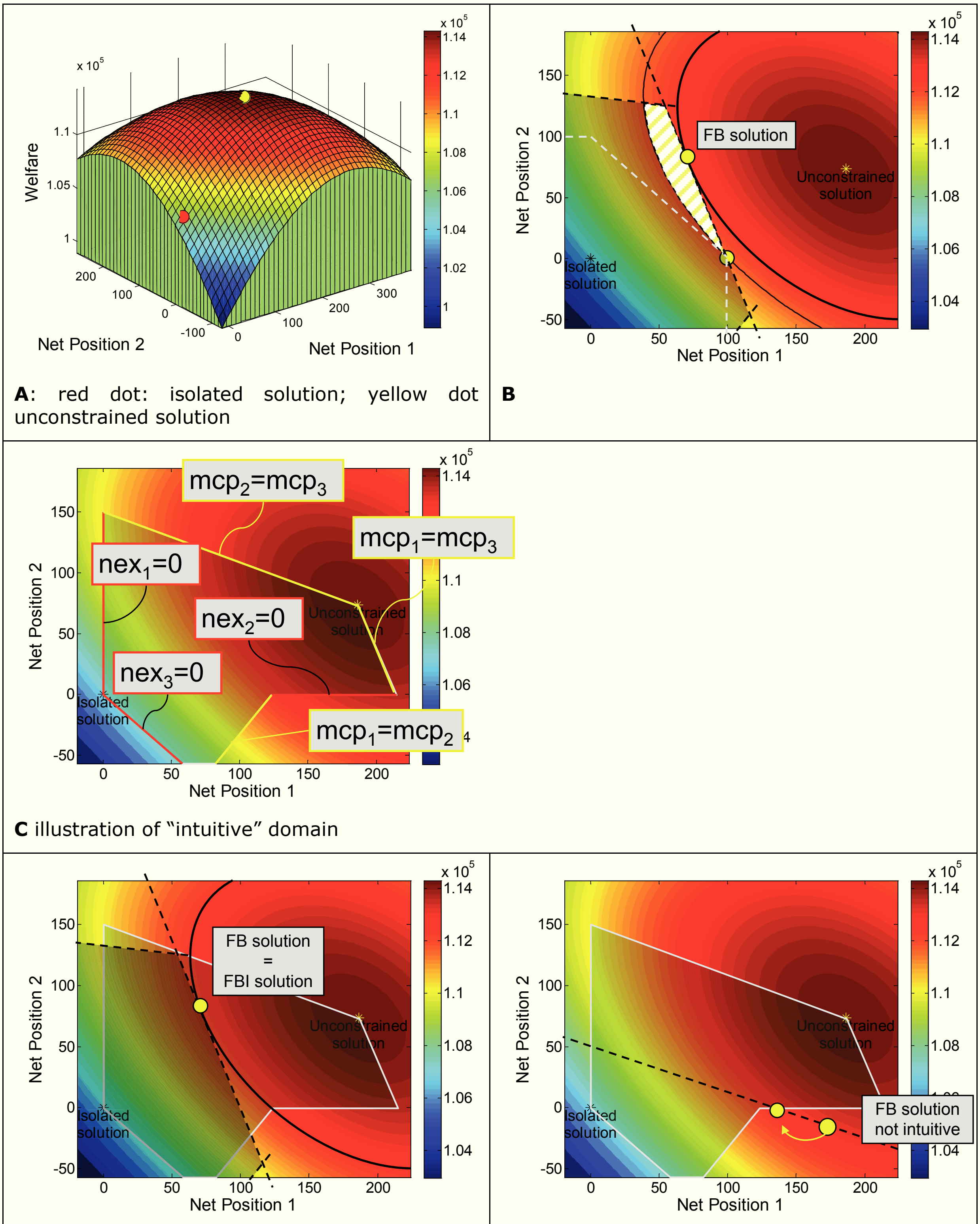
So far no intuitiveness considerations were made. For a (three market) solution to become intuitive, we either need to isolate the non-intuitive market, or to create a partial convergence with one of its neighbours. All these situations are illustrated and form the edges of the "intuitive" domain. For a solution to be intuitive, it must be inside this domain.

Figure D

This illustrates a FB domain where the optimal solution is inside the "intuitive" domain. I.e. the "intuitive" patch is not triggered, and there is no difference between the "plain" solution and the "intuitive" solution.

Figure E

This illustrates a different FB domain (everything below the black dotted line) where the optimal solution is not inside the "intuitive" domain. In order to restore intuitiveness, the "intuitive" patch is triggered, and maps the solution to the highest welfare solution inside the "intuitive" domain. The "plain" and "intuitive" solutions differ, and the "plain" solution yields more welfare.



A: red dot: isolated solution; yellow dot unconstrained solution

B

C illustration of "intuitive" domain

D example FB = FBI

E example FB ≠ FBI

Figure 2 Illustrations of FB and FBI solutions in welfare plots

4 Implementation of intuitive patch

4.1 Intuitive constraints

Rather than explicitly enforcing the "intuitive" domain, Euphemia models "intuitive" constraints that substitute a FB constraint that caused a non-intuitive situation. In section 1 we found that non-intuitive situations stem from the fact that some exchanges relief a congestion, which can then be non-intuitively scheduled, to allow for a more welfare generating exchange elsewhere. In order to prevent non-intuitive situations we discard these relieving effects.

Graphically this is illustrated in Figure 3`. On the left is the illustration of a non-intuitive solution. The red CB is being relieved by the non-intuitive A→B exchange. Discarding relieving effects is illustrated on the right: the CB for A→B exports is substituted by the purple line which discards the relieving effects of A→B exchanges: the line no longer slopes upwards.

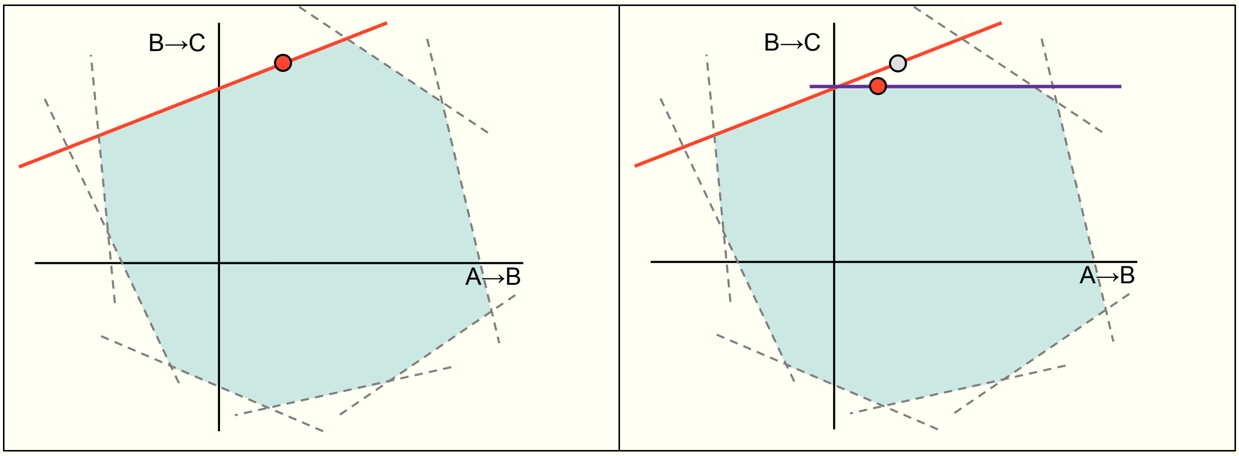


Figure 3 Illustration of an "intuitive" constraint or "intuitive" cut

Analytically the purple line of the above illustration corresponds to substituting the original FB constraint:

$$\sum_{z \in Z} PTDF_z \cdot nex_z \leq RAM$$

By

$$nex_z - \sum_{j \in Z} flowInt_{zj} + \sum_{i \in Z} flowInt_{iz} = 0 \qquad \forall z \in Z$$

$$\sum_{(i,j) \in Z \times Z} \left(PTDF_i - PTDF_j\right)^+ \cdot flowInt_{ij} \leq RAM$$

Where

Z: set of areas;

$PTDF_z$: flow factor for area z;

RAM: remaining available margin of the CB;

$flowInt_{ij}$: Intuitive Flow between areas i and j;

nex_z: net position of area z;

$$(x)^+ = \max(x,0)$$

i.e. we seek a decomposition of the net position (nex_z) into "intuitive" flows ($flowInt_{ij}$). These flows are subjected to the PTDF constraints, but only if $PTDF_i$-$PTDF_j$ > 0 the impact of the flow on the CB is considered. If the flow factor difference is negative, i.e. relieves the CB, this effect is discarded. This modelling therefore is stricter than the original constraint; hence the FB domain becomes smaller. This too was illustrated on the right hand side of Figure 3: the area between the red and purple line no longer is part of the FB domain.

In annex 7.3 of the intuitiveness report[3] it is already explained that this "intuitive" constraint (or "intuitive" cut), might be too strict, and that it could miss an optimal solution that is inside part of the FB domain that is cut off with the new constraint. Therefore the implementation of "intuitive" FB is a heuristic.

This heuristic will work very poorly if the "intuitive" cuts are activated for all CBs at once: the remaining FB domain could be as small as the trivial solution: exchanging zero energy. Instead the "intuitive" cuts are added one at a time. In case a solution is non-intuitive (see section 5.1 on how to determine non-intuitive situations), the CB that is "active" (is constraining the market) and is causing the non-intuitive situation, is substituted by an "intuitive" cut. After adding the "intuitive" cut, it is possible that a new tight PTDF constraint still is causing non-intuitive situations, hence in an iterative fashion further CBs are replaced by "intuitive" cuts until the solution is intuitive. The proof this solution guarantees to result in an intuitive situation follows from the mathematical model, which is presented in annex 5.2.

The iterative process by which "intuitive" cuts are generated is explored in the next section.

4.2 Interaction with block order selection

The mechanism behind block selection in Euphemia is explained in the Euphemia public description[4]. By means of a branch and bound Euphemia traverses the different block and MIC selections, relaxing the fill-or-kill aspects for intermediate solutions, and successively enforcing them until a feasible solution is found. From there the successive iterations are used to improve this solution (in terms of DAMW).

For each block selection, the iterative process by which "intuitive" cuts are generated should be restarted to prove full optimality of the solution. From practice we know that typically once an "intuitive" cut needs to be added, it needs to be added for every block selection. To speed up the algorithm the choice has been made to add the "intuitive" cuts globally (i.e. they apply to the whole B&B tree) rather than locally (i.e. they apply only to the sub tree below the "intuitive" cut). This approach is a further heuristic, but improved algorithmic performance significantly.

[3] http://www.casc.eu/media/CWE%20FB%20Publications/CWE_FB-MC_intuitiveness_report_Oct2013.pdf

[4] Available from all PCR PXs websites, e.g. http://www.apxgroup.com/wp-content/uploads/Euphemia-public-description-Nov-20131.pdf

4.3 Impact on performance

As discussed in the previous sections, the implementation of the “intuitive” patch made some design choices having computational performance in mind at the detriment of optimality:

- The implementation of the “intuitive” cuts that are too strict;
- The activation of these “intuitive” cuts on the whole branch and bound tree, rather than only for local sub tree;

These choices have a theoretical adverse impact on optimality, but make sure that the computational complexity remains manageable.

Since the launch of NWE (5th of February 2014) the Euphemia algorithm is used to run the FB and FBI simulations of the parallel run. The algorithm is configured with the same time constraints as used in the production version. To date Euphemia has always managed to find solutions to both the FB and FBI problems within the production time bounds.

5 Annexes

5.1 Detection of non-intuitive situations

For Euphemia to know whether a solution is already intuitive, or not, and "intuitive" cuts should be generated, Euphemia needs to implement an "intuitiveness" test. The text below described the test used by Euphemia to detect "intuitiveness":

Consider a solution (for a FB balancing area) containing for all areas:

- The net position nex_z;
- The market clearing price mcp_z;

Furthermore a topology has been provided for which the solution needs to be intuitive. The topology TOP describes all pairs of areas (i,j) that should be considered;

STEP 1

Create a graph:

- Use all the areas $z \in Z$ as nodes;
- Create edges for all pairs $(i,j) \in TOP$ for which $mcp_i \leq mcp_j$, i.e. only consider intuitive directions. All edges are associated with infinite capacity;
- Add a source node s and a sink node t.
- Add edges (s,z) for all export areas. Associate capacity equal to the export position;
- Add edges (z,t) for all import areas. Associate capacity equal to the (absolute) import position;

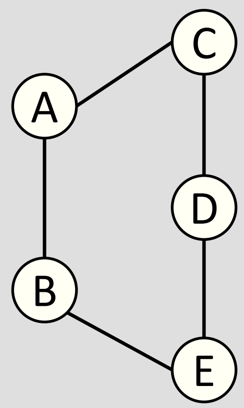
STEP 2

Compute the maximum flow from source s to sink t (using a readily available maximum flow algorithm). If the solution fully saturates all export links that solution corresponds to a feasible intuitive result. If some export capacity remains unused the solution must be non-intuitive.

Example

Imagine 5 markets and the following configuration:

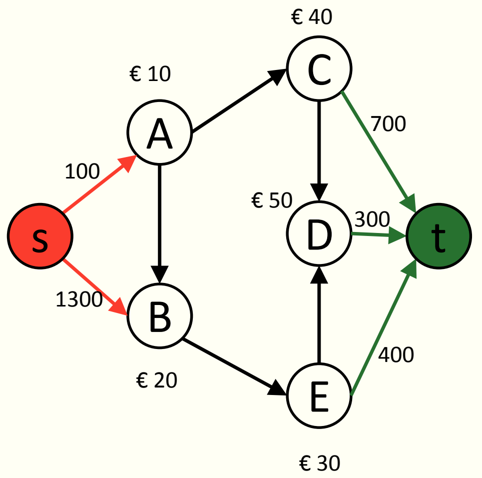
Topology:



Market results (two examples only differ in price):

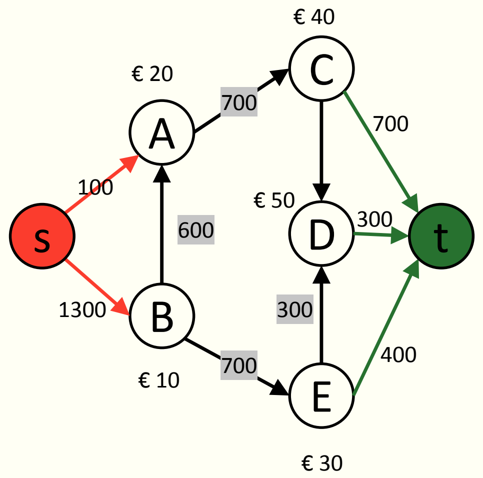
Market	nex	mcp (example 1)	mcp (example 2)
A	100	€ 10	€ 20
B	1300	€ 20	€ 10
C	-700	€ 40	€ 40
D	-300	€ 50	€ 50
E	-400	€ 30	€ 30

Example 1	Example 2
Constructing the graph yields:	Constructing the graph yields:



It is not possible to fully saturate all export capacity from source s: From B there are two directed paths to sink t:

B->E->t and B->E->D-t. These paths only allow 700 of the 1300MW to be exported.



The only difference with example 2 is that the prices of A and B have been inverted; therefore we now have a B->A link rather than an A->B link. Consequently additional routes from B to t have become available:

B->A->C->t and B->A->C->D->t

It is now possible to find intuitive routes exporting all energy (the grey highlighted figures are an example of an intuitive

5.2 FB market coupling model

In the following text the market coupling under FB network constraints is presented. Both the "plain" and "intuitive" models are presented in the underlying mathematical modelling framework. The models presented here focus only on the simple hourly problem and only on the FB constraints. For the interaction with block orders and the other network configurations consult the Euphemia public description[5]. Please note the model presented in this document was previously presented as annex of the feasibility report[6].

Notational conventions

We start by introducing some notational conventions:

Sets

Set	Description	Index
Z	Set of all zones	z
S_z	Set of all sell orders in area z	s
B_z	Set of all buy orders in area z	b
CB	Set of all critical branches (and critical outages)	cb
CB^{FB}	Subset of CBs for which **no** "intuitive cuts" have been added	cb

[5] Available from all PCR PXs websites, e.g. http://www.apxgroup.com/wp-content/uploads/Euphemia-public-description-Nov-20131.pdf

[6] http://www.casc.eu/media/CWE%20FB%20Publications/CWE_FB-MC_feasibility_report_2.0_19102011.pdf

CB^{FBI}	Subset of CBs for which "intuitive cuts" have been added	cb
TOP	Topology on which to enforce intuitivity	(i,j)

Note: by convention $CB^{FB} \cap CB^{FBI} = \varnothing$

Parameters

Parameter	Description
Q_s^z	Quantity of sell order s in area z
Q_b^z	Quantity of buy order b in area z
P_s^z	Price of sell order s in area z
P_b^z	Price of buy order b in area z
$PTDF^{cb}_z$	**P**ower **T**ransfer **D**istribution **F**actor for the influence of zone z on CB cb
RAM_{cb}	**R**emaining **A**vailable **M**argin for CB cb

Variables

Variable	Description	Range	Primal/Dual
x_s^z	Acceptance of sell order s in area z	[0..1]	Primal
x_b^z	Acceptance of buy order b in area z	[0..1]	Primal
nex_z	Net position in area z	$\mathbb{R}$	Primal
nex^{AC}_z	AC net position in area z	$\mathbb{R}$	Primal
$flowInt_{i,j}$	Intuitive flow between areas i and j| (i,j)∈TOP	≥0	Primal
μ_{cb}	Shadow price of CB cb∈CB^{FB}	≥ 0	Dual
π_{sys}	System price	$\mathbb{R}$	Dual
π_z^{Market}	Clearing price related to orders	$\mathbb{R}$	Dual
$\pi_z^{Network}$	Clearing price related to network	$\mathbb{R}$	Dual
$\pi_z^{Intuitive}$	Offset on market price z to make it intuitive	$\mathbb{R}$	Dual
$\mu_{cb}^{intuitive}$	Shadow price of intuitive cut for CB cb∈CB^{FBI}	$\mathbb{R}$	Dual
σ_b	Surplus of buy order b	≥ 0	Dual
σ_s	Surplus of sell order s	≥ 0	Dual

Primal model

Objective function – maximize welfare (cf. annex – Welfare maximization for an explanation)

$$\max \sum_{z\in Z}\left(\sum_{b\in B} Q_b^z \cdot P_b^z \cdot x_b^z - \sum_{s\in S} Q_s^z \cdot P_s^z \cdot x_s^z\right)$$

s.t.

Constraint	Index	Shadow price	ID: Name
$nex_z + \sum_{b\in B} Q_b^z \cdot x_b^z - \sum_{s\in S} Q_s^z \cdot x_s^z = 0$	$\forall z \in Z$	(π_z^{market})	(1) Clearing
$nex_z - nex_z^{AC} = 0$	$\forall z \in Z$	$(\pi_z^{network})$	(2) Export
$\sum_{z\in Z}(nex_z^{AC}) = 0$		(π_{sys})	(3) Balance
$\sum_{z\in Z} PTDF_z^{cb} \cdot nex_z^{AC} \le RAM_{cb}$	$\forall cb \in CB^{FB}$	(μ_{cb})	(4) PTDF
$nex_z^{AC} - \sum_{j\in Z} flowInt_{zj} + \sum_{i\in Z} flowInt_{iz} = 0$	$\forall z \in Z$	$(\pi_z^{\text{intuitive}})$	(5) Intuitive deviation
$\sum_{(i,j)\in TOP}(PTDF_i^{cb} - PTDF_j^{cb})^+ \cdot flowInt_{ij} \le RAM_{cb}$ [7]	$\forall cb \in CB^{FBI}$	$(\mu_{cb}^{\text{intuitive}})$	(6) Intuitive cut
$x_b^z \le 1$	$\forall z \in Z,$ $\forall b \in B$	(σ_b^z)	(7)
$x_s^z \le 1$	$\forall z \in Z,$ $\forall s \in S$	(σ_s^z)	(8)

The clearing constraint (1) relates the accepted order volumes to the net position variables.

The export constraint (2) relates the net position variables to AC net position variables. In this model it is rather superfluous, but in a hybrid coupling that mixes FB and ATC constraints, this contains additional terms relating to the exchanges over the ATC lines.

Intuitive deviation (5) finds a decomposition of (AC) net positions into (intuitive) flows;

Intuitive cut (6) subjects these intuitive flows to the FB constraints. Note that it is a stricter constraint than (4), which it replaces.

The rest of the constraints are self-explanatory.

[7] Where $(x)^+ = \max(x,0)$

Dual model

Objective function

$$\min \sum_{cb \in CB^{FB}} RAM_{cb} \cdot \mu_{cb} + \sum_{cb \in CB^{FBI}} RAM_{cb} \cdot \mu_{cb}^{\text{intuitive}} + \sum_{s \in S} \sigma_s + \sum_{b \in B} \sigma_b$$

s.t.

Constraint	Index	Shadow price	ID+Name
$\pi_z^{market} + \pi_z^{network} = 0$	$\forall z \in Z$	(nex_z)	(9) Price relation
$-\pi_z^{network} + \pi_{sys} + \pi_z^{\text{intuitive}} + \sum_{cb \in CB^{FB}} PTDF_z^{cb} \cdot \mu_{cb} = 0$	$\forall z \in Z$	(nex_z^{AC})	(10) Price coupling
$-\pi_i^{\text{intuitive}} + \pi_j^{\text{intuitive}} + \sum_{cb \in CB^{FBI}} \left(PTDF_i^{cb} - PTDF_j^{cb}\right)^+ \cdot \mu_{cb}^{\text{intuitive}} \geq 0$	$\forall (i,j) \in TOP$	$(flowInt_{ij})$	(11) Intuitive price difference
$Q_b \cdot \pi_z^{market} + \sigma_b^z \geq Q_b \cdot P_b$		(x_b^z)	(12)
$-Q_s \cdot \pi_z^{market} + \sigma_s^z \geq -Q_s \cdot P_s$		(x_s^z)	(13)

The price relation (9) relates market order related prices to network related prices. Since our model is limited to FB only, it is somewhat superfluous, but this way it allows for easier extension to a proper hybrid model (PTDF + ATC). Note that now essentially market and network price are equal (apart from the sign).

Price coupling constraint (10) relates the network price to the shadow prices of the PTDF constraints. Through $\pi_z^{\text{intuitive}}$ and via (11) also the intuitive cuts are taken into consideration. If we substitute (9) in (10) for markets j and i respectively, and take the difference, we get:

$$\pi_i^{market} + \pi_{sys} + \pi_i^{\text{intuitive}} + \sum_{cb \in CB^{FB}} PTDF_i^{cb} \cdot \mu_{cb} = 0$$

$$\pi_j^{market} + \pi_{sys} + \pi_j^{\text{intuitive}} + \sum_{cb \in CB^{FB}} PTDF_j^{cb} \cdot \mu_{cb} = 0 \quad -$$

$$\left(\pi_i^{market} - \pi_j^{market}\right) + \left(\pi_i^{\text{intuitive}} - \pi_j^{\text{intuitive}}\right) + \sum_{cb \in CB^{FB}} \left(PTDF_i^{cb} - PTDF_j^{cb}\right) \cdot \mu_{cb} = 0,$$

Or equivalently:

$\pi_i^{market} - \pi_j^{market} = \left(\pi_j^{\text{intuitive}} - \pi_i^{\text{intuitive}}\right) + \sum_{cb \in CB^{FB}} \left(PTDF_j^{cb} - PTDF_i^{cb}\right) \cdot \mu_{cb}$	(14)

For a $flowInt_{ij} > 0$ the complementary slackness relation dictates that (11) should be hold with equality:

$$\pi_j^{\text{intuitive}} - \pi_i^{\text{intuitive}} = \underbrace{- \sum_{cb \in CB^{FBI}} \underbrace{\left(PTDF_i^{cb} - PTDF_j^{cb}\right)^+}_{\geq 0} \cdot \underbrace{\mu_{cb}^{\text{intuitive}}}_{\geq 0}}_{\leq 0}$$

$$\Rightarrow \pi_j^{\text{intuitive}} - \pi_i^{\text{intuitive}} \leq 0$$

Combining with (14) gives:

$\pi_i^{market} - \pi_j^{market} \leq \sum_{cb \in CB^{FB}} \left(PTDF_j^{cb} - PTDF_i^{cb} \right) \cdot \mu_{cb}$	(15)

Finally constraints (12) and (13) put constraints on the surplus variables. Combined with complementary slackness these state that in-the-money orders should be accepted, and out-of-the-money orders should be rejected.

In section 3.1 it was explained that the "intuitive" cuts are added one by one, which means the problematic CBs move from CB^{FB} to CB^{FBI} in the problem notation. As long as results are non-intuitive, more and more CBs are transferred, until either the solution is intuitive, or $CB^{FB} = \varnothing$. In that case (15) becomes:

$\pi_i^{market} - \pi_j^{market} \leq 0$, i.e. exporting market i will have a price below that of importing market j: results are intuitive.

5.3 Annex - Welfare maximization

The primal objective function is to maximize social welfare, although from the terms in this objective it may not be immediately apparent how this relates to the typical welfare function which is expressed as the sum of the buyer (or consumer) and seller (or producer) surplus and the congestion rents. This section explains this relation.

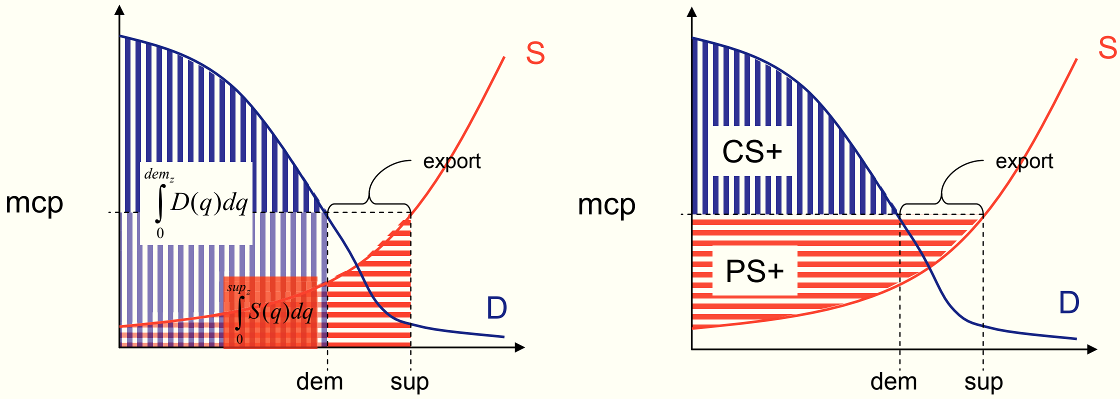


Figure 4

Consider Figure 4 where a supply and a demand curve of a single market are illustrated. The market clears at a price mcp, where supply and demand do not meet: the market exports the difference. The two illustrations contain the same curves. The LHS illustrates the primal welfare function (i.e. $welfare = \sum_{z \in Z} \left(\int_0^{dem_z} D(q)dq - \int_0^{sup_z} S(q)dq \right)$), whereas the RHS illustrates consumer surplus and producer surplus. From the illustration it is apparent that:

$$CS+ = \int_0^{dem_z} D(q)dq - mcp \cdot dem$$, and

$$PS+ = mcp \cdot \sup - \int_0^{sup_z} S(q)dq$$

Therefore CS^+ + PS^+ equals:

$$CS^+ + PS^+ = \int_0^{dem_z} D(q)dq - \int_0^{sup_z} S(q)dq + nex \cdot mcp$$, where nex = sup – dem

Coupling many markets will generate a surplus of:

$$\sum_{z \in Z} \left(CS^+ + PS^+ \right) = \sum_{z \in Z} \left(\int_0^{dem_z} D(q)dq - \int_0^{sup_z} S(q)dq \right) + \sum_{z \in Z} nex \cdot mcp = welfare + \sum_{z \in Z} nex \cdot mcp$$

Shuffling terms:

$$welfare = \sum_{z \in Z} \left(CS^+ + PS^+ \right) - \sum_{z \in Z} nex \cdot mcp = \sum_{z \in Z} \left(CS^+ + PS^+ \right) + CR$$

Why is $-\sum_{z \in Z} nex \cdot mcp$ the congestion rent? Recall nex > 0 means the market exports, or the TSO buys the energy, hence a negative sign: TSO pays money. For nex < 0 the market imports, or the TSO sells the energy, which should have a positive sign: TSO receives money. The positive sign is obtained by negating the sign of the net position.

Under strong duality for optimal solutions the primal and dual objective function are equal. The trained reader will recognize that the new welfare function corresponds to dual objective.